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# Math Interventions for Students with Mild Disabilities: A Meta-analysis and Graphic Organizer Intervention Study

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## ACCEPTANCE

This dissertation, MATH INTERVENTIONS FOR STUDENTS WITH MILD DISABILITIES: A META-ANALYSIS AND GRAPHIC ORGANIZER INTERVENTION STUDY, by JAMES RAYMOND SCHWAB, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education and Human Development, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chair, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty.

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James Raymond Schwab

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- Schwab, J.R., Johnson, Z., Ansley, B., Houchins, D.E., & Varjas, K. (2016). A literature review of alternative school academic interventions for students with and without disabilities. *Preventing School Failure*, 60(3), 194-206. doi: 10.1080/1045988X.2015.1067874 (non-data-based)
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# Math Interventions for Students with Mild Disabilities: A Meta-analysis and Graphic Organizer Intervention Study

by

James Raymond Schwab

Under the Direction of David E. Houchins

## Abstract

Students with emotional behavioral disorders (EBD) who have been removed from their regular schools into alternative educational settings (AES) have academic deficits that affect their success in school (Lehr, Tan, & Ysseldyke, 2009); however, few researchers have investigated what strategies work best for this population, especially in the area of math (Schwab, Johnson, Ansley, Houchins, & Varjas, 2016). Two important areas that students with EBD must master to graduate high school are fractions and algebra (Templeton, Neel, & Blood, 2008). Since the research on math interventions for students with EBD in these areas is limited, researchers have suggested examining the math literature for students with learning disabilities (LD) to find potential intervention components. The purpose of the first study was to synthesize the randomized control trials and quasi-experimental intervention research on instructional approaches that enhance the math achievement of students in grades 6-12 with LD. This study used meta-analytic techniques to synthesize the math literature for secondary students with LD.

Findings indicated that strategy instruction had a higher effect size (Hedges  $g = .72$ ) than alternate delivery systems (Hedges  $g = .23$ ), and the number of Common Core State Standard math practices was a moderator for the effect size of math interventions. Since strategy instruction had a higher effect size, the purpose of the second study was to test the effects of a graphic organizer on the math performance for middle school students with EBD in an AES. This study used a one-group nonequivalent dependent variables design (Shadish, Cook, & Campbell, 2002) with multiple measures in multiple waves to assess the effects of the graphic organizer on the math skills of the students. A repeated measures ANOVA indicated that students significantly improved their math performance on both fractions and algebra using researcher developed measures. Fidelity data indicated that two teachers had low adherence, quality of instruction scores and had low percentages of student engagement. Social validity results indicated that teacher and students found the intervention to be an acceptable intervention.

INDEX WORDS: Math, Learning Disability, Emotional Behavior Disorder, Graphic Organizer, Alternative Schools, Self-regulated Strategy Development, Strategy Instruction



Math Interventions for Students with Mild Disabilities: A Meta-analysis and Graphic Organizer  
Intervention Study

by

James Raymond Schwab

A Dissertation

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in

Education of Students with Exceptionalities

in

Educational Psychology, Special Education, and Communication Disorders

in

the College of Education and Human Development

Georgia State University

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## Terms Page

**Strategy Instruction-** Instruction that included some type of remembering device, graduated instructional sequence, problem-solving strategy, self-instruction, or self-monitoring system to help students learn and recall information.

**Alternate Delivery System-** Instruction that included cooperative learning strategies, changes to placement, technology, or co-teaching.

**Multiple Representations-** Instruction that included representing a math problem in more than one way. This can include modeling the math using manipulatives, drawings or equations.

**Conceptual Understanding-** Instruction that focused on teaching students to recognize the connections and relationships between math ideas.

**System of Equations-** A set of two equations that requires students to solve for two variables such as  $5x = 35$  and  $3x - y = 16$ .

**Binomial Expressions-** Two terms that students must multiply to combine the terms such as  $(x + 4)(x - 3)$ .

**Student Verbalization-** Verbalizing or writing the steps in a solution to a math problem.

**Multi-step Equations-** Equations that require more than two steps in order to solve for a given variable such as  $3(x + 4) = 10$ .



## Chapter 1

### A Meta-analysis of Math Interventions for Secondary Students with Learning Disabilities

The Every Student Succeeds Act (2015) states that schools hold students with disabilities to the same academic standards as their peers without disabilities. The Individuals with Disabilities Improvement Education Act (IDEA; 2006) requires that students with high incidence disabilities have access to the same general education curriculum as their peers. One area in the general education curriculum that is particularly challenging for students with high incidence disabilities is math (Butler, Miller, Crehan, & Babbitt-Pierce, 2003). Historically, these students have made poor progress in math (Gersten et al., 2008). Students must master these standards to graduate high school and in turn get good jobs in the workforce (Powell, Fuchs, & Fuchs, 2013). However, students with learning disabilities (LD) struggle to master math standards (Powell et al., 2013).

#### **Math Struggles for Students with LD**

One potential reason for this struggle to master math standards is that students with LD lack the basic foundational skills (i.e., number sense, fractions, math reasoning) to master more rigorous standards. Students with LD also tend to make simple math calculation errors when solving math problems that require higher order thinking skills (Swanson, Lussier, & Orosco, 2011). In addition, students with LD have difficulty retaining information and solving problems strategically (Gersten et al., 2008). Poor working memory abilities and low cognitive load capabilities may cause students with LD to struggle with comprehending and solving word problems (Swanson et al., 2011). Other possible reasons for the poor math performance of students with LD may be related to teacher factors such as ineffective instruction (Fuchs et al., 2005), inadequate understanding of research-based strategies (Maccini & Gagnon, 2006), or skill focused instruction instead of a conceptual approach (Myers et al., 2015). Given these

conditions, math research on effective instruction for secondary students with LD is an imperative so that teachers can be informed about effective math strategies (Hughes, Maccini, & Gagnon, 2003).

### **Math Instructional Approaches for Students with LD**

Math instructional approaches for students with LD can be classified into strategy instruction, including elements of explicit instruction, and alternate delivery systems (Maccini, Mulcahy, & Wilson, 2007). Strategy instruction is a student-centered approach that teaches students how to learn information and then retrieve that information when it is needed (Swanson, 2001). According to Swanson and Hoskyn, (1998) strategy instruction includes: (a) elaborate explanations (i.e., systematic explanations, elaborations, and/or plans to direct task performance); (b) modeling from teachers; (c) reminders to use certain strategies or procedures; (d) step-by-step prompts or multi-process instructions; (e) dialogue between the teacher and student; (f) questions from teachers; and (g) the provision of teacher assistance when necessary. Alternate delivery system refers to the medium in which the instruction is delivered (Maccini et al., 2007). This can include technology such as computer-assisted instruction and video-based learning (Maccini & Hughes, 1997).

To identify possible effective instructional approaches for students with LD, three syntheses (Maccini & Hughes, 1997; Maccini, Mulcahy, & Wilson, 2007; Myers et al., 2015) and two meta-analyses (Gersten et al., 2009; Zheng, Flynn & Swanson, 2013) were published. The syntheses specifically focused on secondary students with LD. The three meta-analyses (Gersten et al., 2009; Swanson & Hoskyn, 1998; Zheng et al., 2013) focused on different elements of strategy or explicit instruction for students in grades k-12. Zheng, Flynn, and Swanson (2013) synthesized seven group and eight single-case design (SCD) word problem

intervention studies published between 1986 and 2009 for third through eighth grade students with a math LD. The authors examined the math gains between students with a math LD and students with both a math and a reading LD. Additionally, the authors examined whether or not effect sizes varied as a function of specific instructional components. Findings indicated that interventions helped students with LD in math make greater math gains than students with both a math and a reading LD. The authors also found that instructional components that included: (a) sequencing, (b) explicit practice, (c) task reduction, (d) advanced organizers, (e) questioning, (f) task difficulty control, (g) elaboration, (h) skill modeling, and (i) strategy cues significantly improved treatment outcomes for students with LD in math. These instructional approaches are all components of strategy instruction or explicit instruction. In addition, these findings were specific to word problems.

Gersten et al. (2009) conducted a meta-analysis of 42 randomized control trials and quasi-experimental studies of math published between 1971 and 2007 for students with LD in third to eighth grade. In particular, the authors examined the impact of instructional approaches on treatment outcomes for students with LD. They found that the overall effect size for math interventions for students with LD was .63 indicating a medium effect size, but studies addressing older students had lower effect sizes with effect sizes decreasing .07 standard deviations per grade level. Also, the authors found that the following instructional approaches produced significant mean effects: (a) employing explicit instruction (Hedges  $g = 1.22$ ); (b) teaching use of heuristics (Hedges  $g = 1.56$ ); (c) instructing students to verbalize their math reasoning (Hedges  $g = 1.04$ ); (d) using visual representations while solving problems (Hedges  $g = .46$ ); (e) providing students a range and sequence of examples (Hedges  $g = .82$ ); (f) providing ongoing formative assessment data and feedback to teachers and students (Hedges  $g = .21$ ); and

(g) using peer-assisted math instruction (Hedge  $g = .14$ ). One study (Bottge, Heinrichs, Mehta, & Ya-Hui, 2002) was categorized as “other” since it used enhanced anchored instruction, an alternate delivery system intervention (i.e., technology-based interventions such using video discs or other types of medium to deliver instruction)..

Maccini et al. (1997, 2007) synthesized the literature for secondary students from 1988 to 2006 and found 43 intervention studies published. Interventions were coded as (a) behavioral strategies (i.e., instruction that involved demonstration of a skill, modeling, feedback, reinforcement, and teacher directed approaches); (b) cognitive strategies (e.g., strategy instruction); and (c) alternate delivery system. Myers et al. (2015) updated the previous literature reviews (Maccini & Hughes, 1997; Maccini et al., 2007) and examined the literature for secondary students with LD and added 15 studies published between June 2006 and October 2014. Instead of using the previous coding system, the authors categorized studies into (a) cognitive and metacognitive strategies for solving word problems; (b) use of representation to increase conceptual knowledge and problem solving skills; and (c) enhanced anchored instruction. Cognitive, metacognitive, and representation strategies are all different types of strategy instruction (Gersten et al., 2008), while enhanced anchored instruction falls under the category of alternate delivery system (Maccini et al., 2007). These syntheses revealed the types of instructional approaches used in middle school math research for students with LD, but they did not suggest if one approach (e.g., strategy instruction versus alternate delivery system) was better than another for secondary students with LD.

### **Findings on Math Domains**

The National Mathematics Advisory Panel (NMAP; 2008) defined five math domains that students should master including (a) operations, (b) word problems, (c) fractions, (d)

algebra, and (e) general math proficiency. Gersten et al. (2009) used a regression analysis and found that effect sizes for measures of word problems were higher than measures from the other math domains. Zheng et al. (2013) found the range of effect sizes to be higher for word problems than other domains. However, with the introduction of the CCSS (2010), new math domains have been defined. Previous literature syntheses and meta-analyses have not sufficiently taken these more recent math domains into consideration (Powell et al., 2013).

### **Common Core State Standards and Practices**

The CCSS provide a framework for teachers of the necessary math skills for students in grade K-12 that is different from previous math standards (Powell et al., 2013). In sixth and seventh grade, students must master: (a) ratios and proportional relationships; (b) the number system; (c) expressions and equations; (d) geometry; and (e) statistics and probability. Starting in eighth grade, students must also study functions. Finally, in high school, students must also learn number quantity and modeling. Students must be able to perform algorithms to solve each of these standards as well model the math behind these standards. In addition, since the CCSS build from previous standards taught in earlier grade levels, students must have mastered the conceptual knowledge from previous grade levels to understand higher math standards encountered in middle and high school (Powell et al., 2013). For example, students must conceptually comprehend fractions to master algebra standards (NMAP, 2008). Since students with LD must master a number of CCSS (Powell et al., 2013), it is important to examine the number of CCSS addressed in math intervention studies because it may affect the math outcomes.

In addition to the math content students must master, CCSS also published a set of processes and practices that students should become proficient at to meet the CCSS. The CCSS

math practices require students to (a) make sense of problems and persevere in solving them; (b) reason abstractly and quantitatively; (c) construct viable arguments and critique the reasoning of others; (d) model with mathematics; (e) use appropriate tools strategically; (f) attend to precision; (g) look for and make use of structure; and (h) look for and express regularity in repeated reasoning. Students with LD have made minimal progress in using these practices to master math standards (Powell et al., 2013).

### **Potential Moderators**

Using a regression analysis, Gersten et al. (2009) found several potential moderators for math interventions for students with LD in elementary and middle school. They found that (a) design (experimental versus quasi-experimental); (b) control group type (control versus comparison); (c) publication characteristics (peer-reviewed versus dissertation); (d) dependent measure type (posttest versus maintenance versus transfer); and (e) dependent measure design (researcher versus standardized) all moderated the effect size. Specifically, the authors found that studies that used control groups, standardized measures, and transfer measures were associated with smaller effect sizes. However, it is not known if these findings are only applicable with secondary math intervention studies.

### **Rationale and Purpose**

While there are two published meta-analyses for students with LD in grades k-12., an updated meta-analysis to examine the treatment outcomes for secondary students with LD is necessary for four reasons. First, alternate delivery systems instructional approaches for secondary students with LD have not been thoroughly examined (e.g., Maccini & Hughes, 1997; Maccini et al., 2007; Myers et al., 2015). The current study aimed to determine if there is a difference in effect sizes between strategy instruction and an alternate delivery system.

Second, with the implementation of the CCSS (2010), secondary students are required to master more difficult standards to move from grade level to grade level (Powell et al., 2013). Previous research did not specify the CCSS. Gersten et al. (2009) focused on instructional approaches and reported effect sizes between word problems and other domains, but effect size differences between other domains were not reported. Zheng et al. (2013) focused only on word problem instruction, but did not cover other math domains such as fractions. It is unknown if the effect sizes will differ according to different CCSS (e.g., expressions and equations vs. ratios and proportional relationships). Therefore, it is necessary to examine math interventions in terms of CCSS.

Third, the number of math studies including data analysis for students with LD has increased since the Gersten et al. (2009) meta-analysis (Myers et al., 2015). The search date for this meta-analysis ended in August, 2007. For example, Gersten et al. (2009) reported one study in a separate category (Bottge, Heinrichs, Mehta, & Hung, 2002). The authors reported that this intervention showed promise to teach students with LD. Recently, at least six more studies were conducted using this intervention for students with LD (Myers et al., 2015). In addition, Gersten et al. did not include computer-assisted instruction in their meta-analysis. Since technology is becoming more prevalent when teaching students with LD (Bottge et al., 2015), it is necessary to conduct a meta-analysis for secondary students with LD that includes these types of instruction.

Fourth, moderators that can explain the variance in math intervention outcomes should be further explored. Gersten et al (2009) reported effect sizes for moderators including (a) type of design; (b) control group instruction, (c) publication characteristics; (c) measurement characteristics; (d) student grade level, and (e) treatment characteristics (i.e., number of sessions, treatment components, and interventionist characteristics). Zheng et al. (2013) found differences

in word problem intervention outcomes among different math subtypes (i.e., math LD versus reading LD). However, it is not known whether these differences will hold true across all the CCSS.

Previous literature reviews (Maccini & Hughes, 1997; Maccini et al., 2007; Myers et al., 2015) provide findings on the instructional components that have shown positive results for secondary students with LD. However, there is still a need to provide details on which intervention approach is more effective than another for secondary students with LD. Since students with LD must master CCSS to graduate high school (Powell et al., 2013), it is important to examine the literature to determine which instructional approaches are most effective in terms of CCSS. An updated meta-analysis will help identify effective intervention components for teachers to use with students with LD. Therefore, the purpose of this study was to synthesize the randomized control trials and quasi-experimental intervention research on instructional approaches that enhance the math achievement of students in grades 6-12 with LD. The research questions for this study were: (1) Is there a difference in effect sizes between strategy interventions and alternate delivery systems? (2) Is there a difference in effect sizes in terms of terms of CCSS and practices? (3) Do control group type, dependent measure type (posttest vs maintenance) and dependent measure design (researcher vs standardized) influence the efficacy of math interventions on math achievement?

## **Method**

### **Literature Search**

This literature search was comprised of six steps. The first step involved searching relevant electronic databases including: Academic Search Complete, ERIC, Education Full text, PsycARTICLES, PsycEXTRA, and PsycINFO. Multiple combinations of the following



descriptors were used: *math, learning disability, secondary, middle school, disabilities, mathematics disabilities, at-risk, literature review, fractions, number operations, geometry, algebra, data analysis, measurement, probability and high school, learning disabled, meta-analysis, math difficulties, and mathematics difficulties*. Second, the authors followed up the electronic search with a hand search of major journals commonly reporting intervention research for students with LD (i.e., *Learning Disabilities Research and Practice, Learning Disabilities Quarterly, Education and Treatment of Children, Journal of Learning Disabilities, and Remedial and Special Education*). Third, to find unpublished in-press articles, a search of journals offering Online First articles was conducted of the following journals: *Journal of Educational Psychology, Journal of Learning Disabilities, Journal of Special Education, Remedial and Special Education, Exceptionality, Preventing School Failure*, and the *Journal of Education for Students Placed at Risk*, ([www.apa.org/pubs/journals/edu/index.aspx](http://www.apa.org/pubs/journals/edu/index.aspx), [online.sagepub.com](http://online.sagepub.com), and [www.tandfonline.com/page/openaccess](http://www.tandfonline.com/page/openaccess)). Fourth, the authors searched for studies funded by Institute of Education Sciences that were posted on their website ([www.ies.ed.gov](http://www.ies.ed.gov)) in the spring 2017. Fifth, a search was performed of previous literature reviews and intervention studies. Sixth, to find unpublished studies a search was performed through Dissertation and Master's Abstract indexes in ProQuest, Cochrane Database of Systematic Reviews, and relevant conference programs. Additionally, math researchers likely to have conducted work in this area were contacted for information on any additional publications. The first author conducted the initial search. A doctoral student was then trained in search criteria procedures and duplicated the initial search. Inter-rater reliability between the two searches was 100%.

### **Criteria for Inclusion**

This review used the following inclusion criteria: (a) published between fall of 1990 and July of 2016; (b) included a sample of grades 6-12; (c) included means and standard deviations for samples with at least 50% of participants with LD; (d) used at least one academic independent variable; (e) used at least one math dependent variable; (f) employed a design that were group experimental or quasi-experimental; and (g) conducted in the United States. Intervention was defined as instructional practices and activities designed to enhance the mathematic achievement of students with LD (Gersten et al., 2009).

The initial search yielded 2237 studies (1,237 journal articles and 1000 dissertations). The articles were reviewed based on the abstract. Two-thousand one-hundred eighty-nine ( $n = 2189$ ) articles that were either duplicates or not about math interventions were excluded. The remaining articles were closely reviewed using the specific criteria described below. After excluding studies for being single-case designs ( $n = 21$ ) or not providing data analyses for students with LD ( $n = 10$ ), 17 studies remained.

### **Coding Procedures and Inter-rater Reliability**

Coding definitions were discussed between the first author and a doctoral student until consensus was reached. Then, the first author trained a doctoral student in coding procedures and used the coding system to conduct the final coding of all studies independently. Across the total variable matrix, the mean inter-rater agreement was 100%, with the inter-rater agreement values for all codes at 100%.

### **Coding Definitions**

Due to the low number of studies, all variables were dichotomously coded. Math instructional approaches were categorized into strategy instruction and alternate delivery systems (Maccini et al., 2007). Strategy interventions was defined as instruction that included: (a) some

type of remembering device; (b) graduated instructional sequence; (c) problem-solving strategy; and (d) a self-instruction or self-monitoring system to help students learn, organize and recall information. Alternate delivery system was defined as instruction that included: (a) cooperative learning strategies, (b) changes to placement, (c) technology, or (d) co-teaching.

Since previous meta-analyses for students with LD (e.g., Gersten et al., 2009) indicated type of control group as a potential moderator for effect sizes, two types of control groups were coded (i.e., control group and comparison group). Control referred to the control groups who did not receive any training from the researchers during the study. Comparison group was defined as the control groups who received some type of training other than the intended intervention from the researcher during the study. CCSS were coded as a single standard (i.e., the intervention focused on a single standards such as ratios and proportional relationships), or multiple standards such as using both the number system and geometry. The CCSS math practices were defined as either the study's instructional components used at least half of the CCSS math practices (i.e., more than four math practices or less than four math practices). The original coding was to examine each individual CCSS math practice, but due to the low number of studies, this was not possible. Dependent measure design was coded into researcher developed and standardized. Researcher developed measures were defined as measures that were developed by the researchers and standardized measures were defined as measures not developed by the researchers that had been standardized. Dependent measure type was coded into posttest and maintenance and transfer measures. Posttests were defined as any measure that was implemented after intervention instruction began and was aligned to the instruction and similar to the pretests. Maintenance measures were defined as a parallel form of the posttest given two or more weeks after the end of the instructional intervention to assess maintenance of effects

(Gersten et al., 2009). Transfer effects were defined as measures that measured the student's ability to solve math problems they were not exposed to during instruction (Gersten et al., 2009).

### **Analytic Strategy**

Hedges  $g$ , corrected for sample size bias, was used as the measure of effect size since it provides a better estimate of effect sizes than Cohen's  $d$  on small sample sizes (Grissom & Kim, 2005). For studies reporting means, standard deviations, and sample size, the following formula was used:

$$g^u = g \left( 1 - \frac{3}{4(N_T - N_C - 2) - 1} \right)$$

With  $g = \frac{X_T - X_C}{s}$  and  $s = \sqrt{\frac{(N_T - 1)S^2_T + (N_C - 1)S^2_C}{N_T + N_C - 2}}$  in which  $g^u$  is the unbiased estimate of Hedges

$g$ ,  $g$  is Hedges  $g$  as traditionally defined,  $N_T$  is the number of participants in the experimental group,  $N_C$  is the number of participants in the control group,  $X_T$  is the mean of outcome scores for participants in the experimental group,  $X_C$  is the mean of outcome scores for participants in the control group,  $S$  is the pooled standard deviation.  $S^2_T$  is the variance of outcome scores for the participants in the experimental group, and  $S^2_C$  is the variance of outcome scores for the participants in the control group.

Effect sizes of all measures were estimated between the experimental group and the control group. All eligible effect sizes in each study were considered. That is, studies contributed to multiple effect sizes as long as the sample for each effect size was independent. For studies that reported multiple effect sizes from the same sample the statistical dependencies using the random effects robust standard error estimation technique developed by Hedges, Tipton, and Johnson (2010) was used. This analysis allowed for the use of clustered data (i.e., effect sizes nested within samples) by correcting the study standard errors to take into account

the correlations between effect sizes from the same sample. The robust standard error technique required that an estimate of the mean correlation ( $\rho$ ) between all the pairs of effect sizes within a cluster be estimated for calculation of the between-study sampling variance estimate,  $\tau^2$ . In all analyses, the estimated  $\tau^2$  with  $\rho = .80$ ; sensitivity analyses showed that the findings were robust across different reasonable estimates of  $\rho$ .

It was hypothesized that the math literature for secondary students with LD would report a distribution of effect sizes with significant between-studies variance, as opposed to a group of studies attempting to estimate one true effect size so a random-effects model was appropriate for the current study (Lipsey & Wilson, 2001). Weighted, random-effects meta-regression models using Hedges et al.'s (2010) corrections were run with ROBUMETA in Stata (Hedberg, 2011) to summarize effect sizes and examine potential moderators because the Q statistics approach only considers one effect size from each study. ROBUMETA considered all effect sizes from each study, which can be less biased. Meta-regression was used to examine whether types of interventions, control group type, dependent measure type (posttest vs maintenance) and dependent measure design (researcher vs standardized), CCSS (multiple standards vs single standard), and number of math practices (more than four vs less than four) moderate the effects. All moderators were dichotomous and entered directly into the meta-regression model.

Publication bias (i.e., the decision to publish a study based on whether or not there were positive effects) was examined using the method developed by Egger, Smith, Schneider, and Minder (1997) to control for unpublished studies. Publication bias is suggested when the Egger et al. publication bias statistic is significantly less than zero ( $p < .05$ ). The funnel plot (see Figure 1) was further examined for potential publication bias. Specifically, in the absence of publication bias, the studies are distributed symmetrically around the mean effect size. In the

presence of publication bias, it is possible that studies with large-medium sample size may be missing if a few studies are missing in the top and middle of the funnel plot. It is possible that a small number of studies may be missing if a few studies are missing near the bottom (Borenstein, Hedges, Higgins, & Rothstein, 2011). The Funnel plot was also used to detect possible outliers (i.e., really large effect sizes). After removing observed outliers, sensitivity analyses were run to examine the adjusted overall effect size and adjusted moderation effects.

## **Results**

Seventeen studies were included in this meta-analysis, which represented a total of 1,121 participants with LD obtained from 17 independent samples. All the studies subsumed 88 effect sizes (39 pretests and 49 posttests). Overall, compared to the control group, math interventions for secondary students with LD significantly improved math performance with a medium effect size, Hedges  $g = 0.42$ , 95% CI [0.22, 0.62]. Next, moderation analyses were conducted to examine whether instructional approach, control group type, CCSS, dependent measure design, dependent measure type, and number of CCSS math practices used moderated the effects of math interventions for secondary students with LD on math performance. Table 1 presents the most relevant features of these 17 studies.

### **The Influence of Instructional Approach**

Meta-regression analyses with the different instructional approaches as the independent variables (pairwise comparisons) and math performance measures as the dependent variable were conducted. As Table 2 shows, strategy instruction showed higher effects on math performance than alternate delivery systems, Hedges  $g = .74$ , 95% CI [.29, 1.20]

**Table 1***Study Characteristics*

Study	Publication Type	T(n)	C(n)	Instructional Approach	Control Type	Measurement Design	Measurement Type	CCSS	Fidelity	Math Practice
Bottge et al., (2015)	Journal	62	72	Alternate delivery system	Control	Both	Both	Multiple standards	yes	1,2,3,4,5,6,7
Bottge et al., 2013	Journal	146	163	Alternate delivery system	Control	Both	Both	Multiple standards	yes	1,2,3,4,5,6,7
Bottge et al., (2010)	Journal	29	25	Alternate delivery system	Comparison	Both	Both	Multiple standards	yes	1,2,3,4,5,6,7
Butler et al., (2003)	Journal	26	24	Strategy instruction	Comparison	Standardized	Posttest	Ratio and proportional relationships	yes	1,4,5,7
Calhoon & Fuchs, (2003)	Journal	45	47	Alternate delivery system	Control	Both	Posttest	Multiple standards	yes	1,3
Fuchs et al., (1996) Study 1	Journal	13	11	Alternate delivery system	Control	Standardized	Posttest	Multiple standards	No	1
Fuchs et al., (1996) Study 2	Journal	13	11	Alternate delivery system	Control	Standardized	Posttest	Multiple standards	No	1
Fontana (2005)	Journal	17	17	Alternate delivery system	Control	Researcher	Posttest	Multiple standards	No	1
Iseman & Naglieri (2011)	Journal	14	14	Strategy instruction	Control	Standardized	Posttest	The number system	yes	1,2,3
Ives (2007) Study 1	Journal	14	16	Strategy instruction	Control	Researcher	Posttest	Expressions and Equations	yes	1,4,5,7
Ives (2007) Study 2	Journal	5	5	Strategy instruction	Control	Researcher	Posttest	Expressions and equations	No	1,4,5,7
O'Melia & Rosenberg, (1994)	Journal	85	86	Alternate delivery system	Control	Both	Posttest	Multiple standards	yes	1,3

Stultz et al., (2008)	Dissertation	29	29	Alternate delivery system	Control	Standardized	Posttest	The number system	No	1
Walker & Poteet (1990)	Journal	33	37	Strategy instruction	Control	Researcher	Posttest	Below grade level	yes	1,3,4,5
Wildmon et al., (2004)	Journal	28	11	Alternate delivery system	Control	Researcher	Posttest	Below grade level	yes	1
Witzel et al., (2003)	Journal	34	34	Strategy instruction	Control	Researcher	Both	Expressions and equations	yes	1,2,4,5
Witzel, (2005)	Journal	16	16	Strategy instruction	Control	Researcher	Both	Expressions and equations	yes	1,2,4,5
Xin et al., (2005)	Journal	11	11	Strategy instruction	Control	Researcher	Both	Ratio and proportional relationships	yes	1,2,3,4,5,6 ,7

*Note.* 1= make sense of problems and persevere in solving them; 2= reason abstractly and quantitatively; 3=construct viable arguments and critique the reasoning of others; 4= model with mathematics; 5=use appropriate tools strategically; 6= attend to precision; 7=look for and make use of structure.

### The Influence of Multiple CCSS and Math Practices

Since all students must master CCSS and demonstrate proficiency in math practices (Powell et al., 2013), the number of CCSS were examined to determine if using the intervention to teach multiple standards or a specific CCSS influenced the effects of math interventions on math performance. As Table 2 shows, teaching multiple standards versus a single CCSS did not influence the effects of math interventions on math performance,  $\beta = -.22$ ,  $p = .26$ ,  $\tau^2 = .44$ . In addition, the number of CCSS math practices the intervention used (i.e., more than four versus less than four) was examined. Analyses indicated that the number of CCSS math practices influenced the effects of math interventions on math performance for secondary students with LD,  $\beta = .43$ ,  $p = .015$ ,  $\tau^2 = .17$ .

### The Influence of Other Moderators

As Table 2 shows, type of control group did not significantly influence the effects of math interventions on math performance,  $\beta = -.36$ ,  $p = .15$ ,  $\tau^2 = .17$ . However, using a researcher designed measure versus a standardized measure did influence the effects of math interventions



on math performance,  $\beta = .41$ ,  $p = .012$ ,  $\tau^2 = .14$ . Higher effect sizes were found for researcher developed measures than standardized measures. In addition, analyses indicated that type of measure did not influence the effects of math interventions on math performance for secondary students with LD,  $\beta = .08$ ,  $p = .699$ ,  $\tau^2 = .16$ .

### Publication Bias

Since publication bias can be an issue when calculating effect sizes (Egger et al, 1997), the standard errors intervention effect sizes were examined using ROBUMETA to determine whether the standard errors of intervention effect sizes explained the variance in intervention effect sizes. Results showed that the standard errors of intervention effect sizes did significantly explain the variance in intervention effect sizes,  $\beta = 2.76$ ,  $p = .033$ ,  $\tau^2 = .16$ . However, examination of the funnel plot (see Figure 1) revealed that five studies had effect sizes greater than two. After excluding these studies, the standard errors showed that the standard errors of intervention effect sizes did not significantly explain the variance in intervention effect sizes,  $\beta = 1.13$ ,  $p = .09$ ,  $\tau^2 = .09$ .

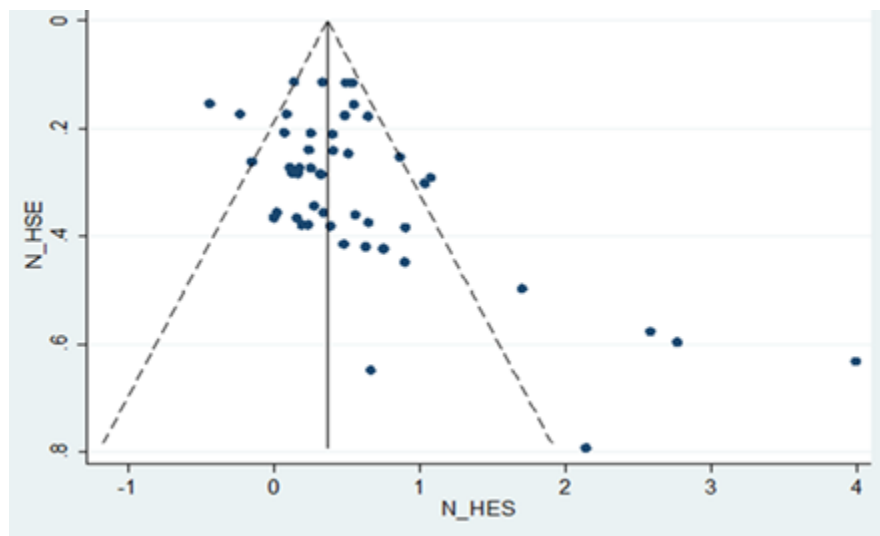
**Table 2**  
*Effect Sizes for Instructional Approach and Moderators*

	k	Coeff	ES	ES/Coeff95%CI	$\tau^2$
<i>Instructional Approach</i>					
Alternate delivery system	24		.23	[.06, .39]	.10
Strategy Instruction	25		.74	[.29, 1.20]	.37
<i>Control Group Type</i>					
Control	31		.31	[.12, .51]	.16
Comparison	17		.69	[.07, 1.31]	.24
CCSS	49	-.23		[-.63, .18]	.19
Multiple Standards	23		.30	[.14, .45]	.11

Single Standard	26		.55	[.14, .97]	.35
<i>Dependent Measure Design</i>	<b>48</b>	<b>.41</b>		<b>[.11, .72]</b>	.14
Researcher	29		.57	[.36, .78]	.12
Standardized	19		.20	[-.14, .55]	.16
<i>Dependent Measure Type</i>	49	.08		[-.37, .53]	.16
Posttest	36		.48	[.27, .69]	.18
Maintenance and Transfer	13		.35	[-.16, .86]	.28
<i>CCSS Math Practices</i>	<b>49</b>	<b>.43</b>		<b>[.10, .76]</b>	.17
4 or more Math practices	35		.61	[.31, .92]	.21
Less than 4 math practices	14		.32	[-.10, .74]	.24

*Note.* k= the number of effect sizes; Coeff= coefficient in the meta-regression model; ES=effect size. Effect size is indexed by Hedges g. ES/coefficient95% CI= 95% confidence interval for effect size or coefficient;  $\tau^2$  = between study sampling variance.

**Figure 1. Funnel Plot for Effect Sizes and Standard Errors**



## Discussion

The purpose of this meta-analysis was to synthesize the randomized control trials and quasi-experimental math intervention research for secondary students with LD. A secondary purpose was to examine possible moderators for the effects of math interventions. The findings suggest that math performance for secondary students with LD can be improved with math interventions. Gersten et al. (2009) found an overall effect size of .62 for elementary and middle school students, but the effect size decreased when age was used as a moderator. The effect size for this study supports this because the effect size was .42 indicating a smaller effect. In addition, it was found that strategy instruction had higher effect sizes than alternate delivery systems. Other meta-analyses have found similar findings that explicit math instruction combined with math strategy instruction is more effective for students with LD (e.g., Gersten et al., 2009; Swanson & Hoskyn, 1998). However, since each study that used strategy instruction had elements of explicit instruction, this meta-analysis did not separate strategy instruction from explicit instruction. Studies conducted by Bottge and colleagues (2010, 2013, 2015) also used some elements of explicit instruction in their alternate delivery systems, but lacked particular strategies to solve the problem-based scenarios. For secondary students with LD, strategy instruction may be a more effective instructional approach since students with LD struggle to act strategically when solving math problems (Gersten et al., 2009).

It was found that control group type, the number of CCSS covered, and the dependent measure type did not influence the effects of math interventions on math performance for secondary students with LD. Gersten et al. (2009) found that studies that had a meaningful treatment in the control group (e.g., comparison group) tended to have statistically significant lower effect sizes than control groups. This meta-analysis was not able to detect a significant

difference in effect sizes for control group type possibly due to a low number of effect sizes. Gersten et al. (2009) also found that interventions with word problems tended to have higher effect sizes than other math domains. Due to the low number of independent effect sizes, the different math domains could not be examined. However, a difference was not detected between multiple CCSS or a single CCSS such as fractions. This suggests that focusing on multiple standards may be just as effective as teaching a single standard (Bottge et al., 2015). Yet, the sequence of instruction may need to be taken into account. For example, teaching unrelated standards such as geometry and probability versus a progression such as fractions to measurement (NMAP, 2008) may have different results in effect sizes. Some authors have suggested teaching fractions when teaching measurement (e.g., Bottge et al., 2015), but the results of this meta-analysis do not recommend one approach versus another.

It was found that the dependent measurement design (i.e., researcher versus standardized) did significantly influence the effects of math interventions. This finding is consistent with previous meta-analyses (Gersten et al., 2009; Swanson & Hoskyn, 1998) who found that lower effect sizes are associated with standardized measures. In this meta-analysis, standardized measures were associated with a small effect size while researcher developed measures were associated with a medium effect size. One explanation for this finding is that researcher developed measures are typically aligned with the instruction so it is easier for students to improve scores. Most studies used a standardized measure as a transfer measure, and students with LD struggle with generalizing strategies to new topics not covered in the initial instruction (Gersten et al., 2008). It is interesting to note a difference was not found between maintenance and transfer measures and posttests. Maintenance measures are closely aligned with the intervention instruction so this affected the effect size of both transfer and maintenance

measures. If the meta-analysis had just included transfer measures, the results may have been different. However, there were not enough independent effect sizes to examine only transfer measures.

This meta-analysis revealed that the number of math practices used in the interventions did influence the effects of math interventions. Using four or more math practices resulted in almost twice the effect (.61) of using less than four math practices (.32). The original intent was to examine using seven math practices versus four, versus three, but due to the low number of studies, this was not possible. Additionally, it is not known whether each of these math practices is equally important. For example, it is not known whether math practice #2 (Reason abstractly and quantitatively) is more important than math practice #4 (model with mathematics). The findings here suggest that more math practices added into instructional interventions may result in higher effect sizes. Since students with LD may require explicit instruction in math processes (NMAP, 2008), it is important to determine if these practices are equally weighted. Swanson and Hoskyn (1998) found that students with LD require explicit instruction in using strategies. For example, to master CCSS math practice # 4 (model with mathematics), students require explicit instruction in modeling math problems (Xin et al., 2005).

It is important to note that 17 studies for secondary students with LD were found through the search procedures. The low number of studies may be due to the excluding of studies conducted for students with LD in elementary schools. In an unpublished study, Schwab et al. found 51 studies conducted for secondary students with LD. However, the majority of studies found were single-case designs. Currently combining single-case design studies with group studies may not be appropriate (Shadish et al., 2015). Single-case designs represent a substantial portion of math interventions for this population (Myers et al., 2015) and could potentially

inform math intervention research. Although, efforts are being made to include single-case designs in meta-analysis (Shadish et al., 2015), exclusion limits the number of math studies that can be included in a meta-analysis. In addition, many group studies did not include data analysis for students with LD. Three studies (Bottge et al., 2010; Butler et al., 2003; Ives, 2007) included at least 50% of participants with a LD and were able to be included in the meta-analysis. The Task Force on Evidence-based Interventions in School Psychology (2006) recommends that studies report subgroup analyses to determine intervention effects on different subgroups. If means, standard deviations, the number of participants in each group are provided for subgroups, then future researchers can use the data in a meta-analysis to determine intervention effects on subgroups.

### **Limitations and Future Directions**

Although a rigorous approach was used to review studies, this meta-analysis is constrained by some limitations. First, although there was a search for unpublished studies (e.g., dissertation and conference articles), most studies included in this present study were peer-reviewed journal articles. Publication bias was not found through the meta-regression with the standard errors. However, the methods for detecting publication bias in meta-analysis are underpowered when there are a limited number of studies (Borenstein et al., 2011). Second, due to the limited number of studies, the coding for each moderator was dichotomous and findings should be interpreted with caution. Dichotomous coding restricts the information that can be gathered from this meta-analysis. Future researchers should examine potential moderators with more than two categories. For example, with regards to the CCSS, it is important to determine if math interventions for secondary students are more effective with fractions versus algebra versus word problems. Examining math interventions in terms of CCSS will help inform researchers of

effective instructional approaches to meet the CCSS, which is an important goal for researchers (Powell et al., 2013). In addition, future researchers should consider the effects of different math practices on math outcomes for students with LD. Third, due to the limited number of independent effect sizes, other potential moderators were not examined. The original intent was to examine the influence of fidelity on math interventions, but there were not enough effect sizes to calculate the mean effect size. Future researchers should examine the role of fidelity on math interventions as well as other potential moderators (e.g., LD subtype, age, duration of intervention, instruction provider).

## **Conclusion**

If educators are to help secondary students with LD meet the new math CCSS, they will require effective interventions. Previous literature reviews have identified some potential intervention components, but it was not known if one approach is more effective than another. The current meta-analysis is a first step in identifying effective intervention components for this population. The findings in this case revealed that strategy instruction had higher effect sizes than alternate delivery systems. However, more research is needed to determine if this is true. With more researchers conducting high quality research for secondary students with LD, it may be possible to help them master difficult CCSS, which in turn will help them to graduate and obtain better employment.

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## Chapter 2

### Using a Graphic Organizer to Increase Math Performance for Students with Emotional Behavioral Disorders in Alternative Schools

Recent legislation for student with disabilities requires them to master the same academic standards as their peers without disabilities (The Every Student Succeeds Act, 2015). In addition, they must have access to the same general education curriculum as their peers (The Individuals with Disabilities Improvement Education Act, 2006). One area in this general education curriculum that can be particularly challenging is math (Butler, Miller, Crehan, & Babbit-Pierce, 2003). Math instruction is even more complex due to reforms within the National Council of Teachers of Mathematics' (NCTM, 2000) and the recent common core state standards initiative (CCSS, 2010). Both these reforms require a focus on conceptual understanding (i.e., demonstration of how the math works) and procedural understanding (i.e., demonstration of following algorithms to solve problems). One group that struggles with math are students with emotional behavioral disorders (EBD; Mulcahy, Maccini, Wright, & Miller, 2014; Ralston, Benner, Tsai, Riccomini, & Nelson, 2014; Templeton, Neel, & Blood, 2008).

Students with EBD often struggle behaviorally and academically in the general education classroom (Reid, Gonzalez, Nordness, Trout, & Epstein, 2004) and are much more likely to drop out of school than any other disability category (Wagner et al., 2006). They often (a) perform several grade levels below their peers (Ralston et al., 2014); (b) show less progress in academics across grade levels (Wagner et al., 2006); (c) exhibit low levels of on-task behavior and task completion (Haydon et al., 2012); (d) lack self-regulation skills (Levendoski & Carledge, 2000); and (e) lack academic skills and content when compared to the same aged peers (Reid et al., 2004). In the school, students with EBD have difficulties attending to instruction, relating new

information to what is already known, and establishing productive work environments (Carr & Punzo, 1993). Many of these students struggle to act purposefully and strategically for their academic benefit and do not manage their own academic behavior in the school setting (Levendoski & Cartledge, 2000). These struggles cause students with EBD to have low academic performance especially in math (Reid et al., 2004).

### **Math Progress for Students with EBD**

Using national data collected from Special Education Elementary Longitudinal Study (SEELS) and the National Longitudinal Transition Study-2 (NLTS2), Wagner et al. (2006) compared scores for students with EBD on the Woodcock Johnson III (Woodcock, McGrew, & Mather, 2001) math computation subtest between elementary school students and high school students with EBD. They found that scores declined across grade levels with the average dropping from the 34<sup>th</sup> percentile ranking among the elementary school students to the 28<sup>th</sup> percentile for students in high school. In a random sample of k-12 students receiving special education services for EBD, Nelson, Benner, Lane, and Smith (2004) found that 56% of children with EBD ages 5 through 12 years old scored below the norm on the Woodcock Johnson III math achievement subtests, while 83% of adolescents ages 13 years old and older scored below the norm. In a longitudinal study, Greenbaum and Dedrick (1996) reported that 93% of students with EBD ages 12 to 14 years of age performed below grade level in mathematics. One possible reason for these low math scores may be due to the fact that students with EBD struggle with attaining and retaining basic and computational math skills such as addition, subtraction, multiplication and division (Templeton et al., 2008). The lack of basic math skills may lead to more school failure and may lead to failure later in life (Hodge, Riccomini, Buford, & Herbst, 2006). In order to facilitate more positive secondary and post-secondary outcomes for students

with EBD, instructional strategies to improve math performance are imperative. This is particularly relevant for students in alternative education schools (AES).

### **AES and EBD**

Students with EBD may display such high levels of inappropriate behavior and low academic skills that they require placement in an AES specifically focused on behavior (Wilkerson, Afacan, Perzigian, Justin, & Lequia, 2016). AES focused on therapeutic services can benefit students with EBD by providing mental health services in addition to special education services (Gagnon & Leone, 2006). According to Tobin and Sprague (1999), the characteristics of AES that may benefit students with EBD may include: (a) a low student to teacher ratio, (b) a highly structured classroom, (c) flexible scheduling, (d) positive behavior management, and (e) individualized instruction. These characteristics can help facilitate a more positive learning environment for the specialized learning needs of students with EBD (Carver et al., 2010; Lehr, Tan & Ysseldyke, 2009; Ruzzi & Kraemer, 2006). However, the academic needs of youth with EBD in AES are one of the most neglected areas in practice and research (Carver, et al., 2010; Lehr et al., 2009; Schwab, Johnson, Ansley, Houchins, & Varjas, 2016).

### **Research on Math and EBD**

Although, the number of research studies addressing academic instruction, particularly math to students with EBD in AES, is limited (Mulcahy et al., 2014; Ralston et al., 2014; Schwab et al., 2016), some key findings have emerged. For students in AES, Schwab et al. (2016) in their systematic review of academic intervention research found that both explicit instruction and self-regulation interventions resulted in positive gains in academics. This aligns with previous math research for students in EBD not attending AES (Gunter, Coutinho, & Cade, 2002; Mulcahy et al., 2014; Ralston et al., 2014). Also, self-regulation and self-management



techniques including strategy instruction have indicated positive effects in math (Mooney, Ryan, Uhing, Reid, & Epstein, 2005; Mulcahy et al., 2014; Ralston et al., 2014).

Two understudied areas for students with EBD are fractions and algebra (Mulcahy et al., 2014; Ralston et al., 2014; Schwab et al., 2016). The National Mathematic Advisory Panel (2008) suggested that students in middle school must master fractions, as they are foundational for success in algebra (Booth & Newton, 2012). Algebra skills are required for students with EBD to graduate high school (Booth & Newton). Therefore, instructional strategies to increase both fraction and algebra performance for students with EBD are needed. Since there are few studies for students with EBD, researchers have suggested that the instructional research literature for students with learning disabilities (LD) may provide some help in identifying other potential intervention components for students with EBD (Hodge et al., 2006).

### **Math Instruction for Students with LD**

**Strategy instruction.** Strategy instruction refers to teaching students a series of steps to follow independently in solving a problem or achieving an outcome (Coyne, Kame'enui, & Simmons, 2001). Strategy instruction requires that the teacher demonstrates a step-by-step plan for solving a specific set of problems and asks students to use the same procedure/steps demonstrated by the teacher to solve the problem (Gersten et al., 2009). In a meta-analysis of 17 math intervention studies for secondary students with LD, Schwab, Houchins, Peng, and Varjas (2017) found that strategy instruction had a higher effect size (Hedges  $g = .74$ ) than alternate delivery systems (Hedges  $g = .23$ ). One type of strategy instruction that has shown positive outcomes for students with LD is teaching students to create multiple representations of a math problem (Myers, Jun, Brownell, & Gagnon, 2015). These can include representing a math problem with concrete manipulatives or drawings (Gersten et al., 2008). Multiple

representations have been used with middle school students with LD to increase math performance in both fractions (Butler et al., 2003) and algebra (Scheuermann, Deshler, & Schumaker, 2009; Witzel, 2005; Witzel, Mercer, & Miller, 2003). Using drawings can help foster conceptual knowledge (Butler et al., 2003), which is a key component of the CCSS (2010). Teaching students a strategy such as using multiple representations may serve as a cue to help students with EBD self-manage as they work math problems (Sawyer, Graham, & Harris, 1992).

**Graphic organizers.** In addition to multiple representations, graphic organizers have shown positive outcomes for students with LD in algebra (Dexter & Hughes, 2011). Graphic organizers have been often been recommended to aid students with LD to understand abstract concepts (Dexter & Hughes, 2011). Graphic organizers are visual arrangements of words, phrases, and sentences, and can include elements such as arrows, and boxes (Ives, 2007). Graphic organizers have been shown to produce positive math gains when teaching students with LD a procedure to solve systems of equations (Ives, 2007) and to assist them with accurately lining up an equation (Strickland & Maccini, 2013). Using a graphic organizer may help students with EBD organize their thinking and in particular organize the different representations of a fraction and algebra problem due to their struggles to manage their own academic behavior and maintain a productive work environment (Mooney et al., 2005).

**Student verbalization.** A third strategy that has shown effectiveness in teaching algebra to students with LD is student verbalization of their math thinking (Gersten et al., 2008). Student verbalization or self-explanation allows students with LD to think through the math steps needed to solve a problem and may aid them in memorizing the steps to solving a specific type of math problem (Gersten et al., 2008). A self-explanation is defined as verbalizing or writing steps in a solution format (Gersten et al., 2008). Studies have used shown self-verbalization to be effective

for elementary (Ross & Braden, 1991; Tournaki, 2003) and secondary students (Hutchinson, 1993). Hutchinson (1993) used a single case multiple baseline to study the effects of teaching 20 middle school students to use self-check statements to solve algebra word problems. Self-check statements included students writing statements such as “Have I written the equation, or Have I got the whole picture for this? Hutchinson found that students with LD substantially improved their scores on word problem measures including maintenance and transfer indicating that allowing students to record the steps used to solve aid in memory. Students with EBD struggle with self-management (Mooney et al., 2005) and require supports to maintain productive academic behavior. Using student verbalization may help them to follow the steps to solving a fraction and algebra problem and remain on-task.

### **Self-regulated Strategy Development**

While the intervention research for students with EBD is limited (Templeton et al., 2008), the self-regulated strategy development (SRSD) academic instructional framework has shown potentially to help students with LD in math (Case, Harris, & Graham, 1992; Cuenca-Carlino, Freeman-Green, Stephenson, & Hauth, 2016). The SRSD model includes procedures for goal setting, self-monitoring, self-instruction, and self-reinforcement (Case et al., 1992). SRSD consists of six stages where educators and/or students including (a) develop the background knowledge, (b) discuss the strategy, (c) model the strategy, (d) memorize the steps, (e) support student use, and (f) facilitate independent performance from the student (Cuenca-Carlino et al., 2016). Researchers have shown that students with EBD require (a) explicit and clear instruction; (b) material presented in a structured and systematic fashion; (c) daily review of previously learned concepts; (d) sufficient supports provided in the early stages of learning; (e) high levels of opportunities to respond to ensure maximum student engagement; and (f) repeated practice

opportunities (Billingsley, Scheuermann, & Webber, 2009). The SRSD framework meets each of these requirements.

### **Self-efficacy**

In addition to specific academic instructional strategy instruction, one area that has shown an impact on the math performance for students with LD is self-efficacy (Cuenco-Carlino et al., 2016). Self-efficacy can be defined as a perceived awareness of one's ability to produce the desired results for a particular task (Bandura, 1997; Zimmerman, 2000). Bandura (1997) proposed that four sources can contribute to self-efficacy: (a) personal achievement and past experiences of success or failure; (b) vicarious learning or learning by watching others (e.g., teachers and students); (c) social persuasion or peer influence; and (d) emotional or psychological states, such as depression or anxiety. Siegle and McCoach (2007) conducted a study in 15 upper elementary school whole-classroom environments to investigate mathematics and self-efficacy. Results indicated the importance of strategy instruction in increasing student self-efficacy. Specifically, instructional strategies that included: (a) lessons clearly modeled by the teacher; (b) review from previous lessons; (c) clear and recursive practices; and (d) focused attention on reviewing each day's lesson led to higher high levels of student self-efficacy in math. In addition, in a qualitative study by Usher (2009), student semi-structured interviews with eight middle school mathematics students indicated that self-regulated strategies and teacher structures were critical factors in student self-efficacy. Students with EBD in AES may have experienced low self-efficacy due to repeated school failure (Lehr et al., 2009), which can affect their math performance.

### **Purpose**

The math intervention research for students with EBD on important math topics such as fractions and algebra is limited (Mulcahy et al., 2014). Since students with EBD are several grade levels below in math (Ralston et al., 2014), math strategies to improve math performance are necessary. Students with EBD struggle with self-regulation (Levendoski & Cartledge, 2000) and require supports to organize their math thinking. Using a graphic organizer may help them organize different representations of a math problem. In addition, using self-check statements may help them complete all steps to solve a math problem. To master CCSS (2010), students with EBD require mastery of conceptual and procedural knowledge (Powell, Fuchs, & Fuchs, 2013). Researchers have found that graphic organizers (e.g., Ives, 2007), multiple representations (Gersten et al., 2009), and student verbalization (e.g., Hutchinson, 1993) can have positive outcomes for students with LD, but it is not known how effective these strategies will be for students with EBD. Therefore, the purpose of this study was to test the effects of a graphic organizer on fraction and algebra performance for middle school students with EBD in an AES. This study sought to answer the following research questions: (1) Does strategy instruction in the use of a graphic organizer combined with self-verbalization to solve fraction and algebraic equations influence student math outcomes in fractions and algebra, including maintenance as measured by researcher developed assessments? (2) Does strategy instruction in the use of a graphic organizer combined with student verbalization to solve fraction and algebraic equations influence student math overall ability as measured by a standardized assessment (e.g., KeyMath-R)? (3) Do middle school teachers of students with E/BD in alternative schools implement the instruction to solve fraction and algebraic equations with fidelity?, (4) Do middle school teachers and students with E/BD in alternative schools find the instruction to solve fraction and algebraic equations to be a socially acceptable intervention?, and

(5) Does strategy instruction in the use of a graphic organizer combined with student verbalization influence self-efficacy as measured by a student survey?

## **Method**

### **Setting**

Students were selected from two public AES that offered therapeutic services for students with behavioral and mental health issues in the southeastern United States. School A was for students in grades k-12 with all students receiving special education services. School B was an AES within a regular education middle school where less than 10 students received supports in resource classrooms, but attended extracurricular activities with their regular education peers. Across the two schools, three classrooms were used. The schools provided comprehensive special education services and therapeutic supports for students diagnosed with disabilities including EBD. The majority of students were classified as EBD, but approximately 100 students with autism are served at the school. Approximately 400 students annually attended the K-12 school, ages 3 to 21 (1% Asian/Pacific Islander, 6% Hispanic, 76% African-American, 15% White, 2% Multi-racial). Instruction took place in the middle school math classrooms. Classrooms typically contain less than 10 students with one teacher and up to two paraprofessionals. At school A, students remained in one self-contained classroom with one teacher teaching the academic subjects of reading, math, social studies, and science. Students also attended elective classes such as music and physical education. At school B, at first students remained in one classroom, but midway through the study, students rotated between the academic subjects with one teacher teaching language arts and social studies and the other teacher provided math and science instruction. Classes were approximately one hour in length with intervention instruction occurring for approximately 30-45 minutes three times per week.

## Participants

**Student participants.** Fifteen middle school students across the three classrooms were recruited. Students were selected based on the following criteria: (a) had a history of math difficulty as defined by classroom teacher and math goals in the Individualized Education Plan, (b) had EBD or challenging behaviors, (c) scored below 50% on the fraction and algebra pretests, (d) scored above 80% on a computation measure using a calculator, (e) returned signed consent form from parent, and (f) signed the assent form. Since instruction took place classwide, all students were considered for inclusion in the intervention study. The research design allowed for each individual participant's data to be compared to his/her previous data. Therefore, students were not excluded from the study based on academic achievement. Students were excluded from data-analyses if they refused to participate in the intervention instruction or assessments ( $n=2$ ) and or if they were withdrawn before data collection of all phases were complete ( $n=2$ ). To be included in the study, consent was obtained from parents and assent was obtained from students. Teachers provided researchers with demographic data on each student using the form included in Appendix B. Eleven students completed the study except one student did not complete the maintenance measure.

**Teacher participants.** Participating teachers were five special education teachers certified to teach math at the middle school level. Teachers A and B were recruited at the start of the study and provided instruction to their own respective classes. At School B, Teacher C provided intervention instruction to his class for the first two lessons until a change in schedule required students to move from remaining in one class to rotating between teachers. Therefore, teacher D was recruited and provided the remaining intervention instruction to the students. Teacher D provided the intervention instruction, but had to leave before administering all the

measures so teacher E was recruited to provide review sessions and administer the remaining posttests. Teachers varied on key demographics such as race and years of teaching experience.

See Tables 3 and 4 for participant demographics.

**Table 3**

*Student Demographics*

	Teacher A	Teacher B	Teacher C,D,E	Total
Total	3	4	4	11
Age				
11	2	0	0	2
12	0	2	3	5
14	0	2	1	3
15	1	0	0	1
Gender				
Male	2	3	4	9
Female	1	1	0	2
Race				
Black or African American	3	2	4	9
White	0	2	0	2
Grade				
6th	2	1	2	5
7th	0	3	2	5
8th	1	0	0	1
Primary Disability				
EBD	3	4	2	9
OHI	0	0	1	1
Autism	0	0	1	1
Secondary Disability				
EBD	0	0	1	1
LD	0	1		1
LI	0	0	1	1
OHI	0	0	1	1
IQ				
Mean scores	90.67	78.25	69.00	
Standard	17.24	2.22	5.60	



deviation

Range	72-106	76-81	61-74	
# on free/reduced lunch	3	4	4	11
# with Math goal in IEP	3	4	4	11

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*Note.* The IQ score was taken from Woodcock Johnson Test of Cognitive Abilities

**Table 4**  
*Teacher Demographics*

Variable	Teacher A	Teacher B	Teacher C	Teacher D	Teacher E
Age	50	52	60	30	47
Gender	Female	Female	Male	Female	Female
Race	African-American	African-American	African-American	African-American	White
Highest degree earned	Master's	Master's	Ed.S	Bachelor's	Master's
Number of years teaching	3	19	36	2	4
Number of years teaching in a therapeutic setting	3	19	15	1	.17
Certification	Special Education	Special Education	Special Education	None	None

## Materials

**Intervention lesson materials.** All materials needed for teaching the graphic organizer lessons were photocopied and placed in a binder for each class period for each teacher. Lessons were based on a modified SRSD framework (Cuenco-Carlino et al., 2016). The researcher provided the teachers with scripted lessons to increase the fidelity of instruction. Instruction was divided into 10 lessons total. Five lessons were provided on fractions computation and five

lessons were provided on algebra equations. Each lesson was divided into approximately 30 minutes of instructional activities each day. For each session, the teacher received (a) a formal lesson plan, (b) all teacher materials for the lesson, and (c) all student materials for the lesson. Teacher materials included dry erase markers and a laminated graphic organizer chart. Teachers used the laminated graphic organizer to work problems for each lesson. Student materials included blank graphic organizers and math worksheets for each lesson. See Appendices C and D for lesson plans along with needed materials.

**Assessment materials.** A variety of measures were collected on students for the purposes of obtaining descriptive data as well as measuring responsiveness to the intervention. All of these materials were provided to the teachers at pre- and post-assessment time points. At each assessment point, teachers were provided with enough copies for their class. Full descriptions of the assessments are provided for each assessment in the dependent variables section below.

## **Training**

**Teacher training.** The teachers were trained by the primary investigator for a total of six hours divided into two three-hour trainings with the first training focusing on fraction instruction and the second training focusing on algebra. Teachers at school A attended the trainings together while at school B, Teacher C was trained with one teacher, who left before the instruction started. Teachers D and E were trained individually after the first teacher left and the schedule was changed. The training followed the practice-based professional development outlined in McKeown, Fitzpatrick, and Sandmel (2014). First, the participating teachers shared group aspects about themselves, their views on math, and student strengths and weaknesses in relation to fractions and algebra. Teachers were asked to share their concerns about fractions and algebra. Second, the rationale for SRSD and graphic organizer was briefly explained. Following

these two foundational activities, teachers were then asked to examine the provided teacher materials in a binder. Third, the researcher modeled teaching lessons one and two to the participants using the same materials that would be used in the classroom. After the lesson was concluded, participants were then asked to teach each other in the same manner. Teachers then received feedback from the trainer. The researcher used a fidelity checklist (see Appendix G) to ensure that all components were completed. By the end of the professional development, the participants had read the lessons, observed them being performed by the trainers, observed sample lessons as taught by another participant, and performed each lesson themselves. The teachers demonstrated 100% on all steps of the adherence checklist and completed the training.

**Data collector training.** Two special education doctoral students were trained on student engagement observational procedures and the use of all fidelity and quality of instruction checklists. They also were trained by the primary investigator to score all math assessments. Once trained, research staff scored all protocols with a minimum of 30% of protocols rescored by a second research staff member independently to calculate inter-observer agreement.

### **Dependent Variables**

**Prescreening measure.** The Monitoring Basic Skills Progress Basic Math Computation (MBSP; Fuchs, Hamlett, & Fuchs, 1998) assessment was administered to the students to determine eligibility in the study. The MBSP is a curriculum-based measurement for grades 1 to 6 that provide a sampling of basic math computation skills ranging from whole numbers to fractions. The third grade form was selected because it measures the basic addition, subtraction, multiplication, and division computational operation skills that students will need to be able to perform with a calculator to solve the fraction and algebra problems. The test has a reliability coefficient range of .94-.98 and a criterion validity median coefficient score of .82 for students

with disabilities (Fuchs et al., 1998). The third grade form included 25 items that measured the subskills of: (a) addition with regrouping, (b) subtraction with regrouping with no zeroes, (c) subtraction with regrouping with zeroes, (d) multiplication basic facts, (e) multiplication with regrouping, and (f) division of basic facts. A percentage score was calculated by taking the number of problem solved correctly divided by the total unnumber of problems.

**Standardized measure.** The *KeyMath-3Revised: A Diagnostic Inventory of Essential Mathematics (KeyMath3-R; Connolly, 1998)* was used as the standardized math measure. The *KeyMath3-R* is a content-referenced test designed to assess student understanding and application of important mathematics concepts and skills. The assessment is available in two parallel forms, designated as Form A and Form B, each of which contains 372 full-color test items grouped into 10 subtests that represent three general math content areas: Basic Concepts, Operations, and Applications. Eight subtests were administered with a flip easel, and two subtests are administered with a Written Computation Examinee Booklet. The *KeyMath3-R* has a validity score for middle school students ranging from .92-.98 and an internal consistence reliability score ranging from .89-.97. It includes 13 mathematical domains (e.g., numeration, rational numbers, geometry) organized into three areas (basic concepts, operations, and applications). The numeration and algebra tests were used for this study because they measured the skills closest to the areas targeted in the intervention. The raw and scaled scores were recorded, but the scaled score was used for analyses in this study. The *KeyMath3-R* was administered before students took the fraction and algebra measures and after all instruction had concluded.

**Researcher developed fraction measure.** Nonstandardized measures were eight problem fraction computation quizzes involving addition and subtraction created by the

researcher and then examined by a math expert. The measure included four problems measuring conceptual knowledge and four problems measuring computation. For validity purposes, a math expert was given a copy of each measure and confirmed that all the probes measured the skill of fraction computation. Scores for each probe were calculated by dividing the number of digits answered correctly by the total number of total number of digits possible to obtain a percentage score. Measures were not counter-balanced, but steps were taken to increase the reliability. These steps included the math expert ensuring that fraction problems did not contain denominators greater than 5. The wording for each problem was the same with only different numbers being changed. The primary investigator scored all the assessments while for each test wave, a second person scored 6 (40%) randomly selected tests. Inter-observer agreement between scorers during all tests for fractions was 100%. See Appendix E for a sample fraction probes.

**Researcher developed algebra measure.** Eight problem two-step variable equations quizzes involving solving equations were created by the researcher and then examined by a math expert to confirm that the quizzes measured the skill of algebra equations. The measure included four problems measuring conceptual knowledge of equations and four problems measuring the ability to solve variable equations. For validity and reliability purposes, the math expert confirmed that all the probes measure the skill of solving variable equations and all questions on each measure used the same wording with different numbers being used. Scores for each probe were calculated by dividing the number of digits answered correctly by the total number of total number of digits possible to obtain a percentage score. The primary investigator scored all the assessments while for each test wave, a second person scored 6 (40%) randomly selected tests.

Inter-observer agreement between scorers for all algebra measures was 100%. See Appendix F for sample algebra probes.

**Social validity questionnaire and interview.** To examine the social validity of using the graphic organizer, the students and teachers were administered the Treatment Acceptability Rating Form–Revised (TARF-R; Reimers & Wacker, 1992). The TARF-R is a brief 20-question seven-point Likert scale assessment. Teachers were asked the entire 20 questions, while the student version was modified to include 10 questions (see Appendices I and J). Means and standard deviations were reported. Interview questions were created to determine participant preferences about their favorite parts of the graphic organizer instruction as well as suggestions for improvement. In addition, questions were created to determine if participants had a preference for algebra versus fraction graphic organizers. Questions were created by the primary investigator and then examined by the research team to check for ease of comprehension (see Appendices K and L).

**Treatment fidelity.** Checklists were created to measure adherence, quality of instructional delivery, and student engagement (Dane & Schneider, 1998). The adherence checklist was based upon the SRSD intervention instruction. The checklist guided teachers through the phases of SRSD as well the specific steps required to solve fraction and algebra problems. For fractions, teachers were required to (a) draw each fraction; (b) demonstrate creating equivalent fractions; (c) demonstrate adding or subtracting the numerators and keeping the same denominator; and (d) demonstrate writing down the steps to solving the problem. For algebra, teachers were required to (a) draw the equation, (b) demonstrate using each inverse operation, and (c) demonstrate writing the steps to solving an algebra equation. Each lesson had a set of planned steps that the teacher should complete, but varied from lesson to lesson (see

Appendix G). As the teacher completed each step that was planned, observers checked off each component. In addition, a math quality of instruction form was created based upon the work of Hill et al. (2008). The quality of instruction form required teachers to (a) use the math vocabulary consistently; (b) perform the math correctly or self-correct mistakes; and (c) call on a variety for students (more than two) to answer questions. Each criteria was rated on a scale of 1 to 3 (see Appendix H).

**Student engagement.** Student engagement was measured using the method developed by Sutherland, Wehby, and Copeland (2000). A momentary time sample procedure with one-minute intervals was used to measure students' on-task behavior. The classroom was divided into three quadrants, with each group of students representing one quadrant. Student engagement was defined as orientation by the target students toward the appropriate objective or person. This behavior included: (a) following direction given by the teacher, (b) paying attention to the speaker (teacher or peer), and (c) working assigned tasks. If any of the students in the observed quadrant during the time sample did not demonstrate the any of the above criteria for student engagement, then the observers recorded not engaged for that interval.

**Self-efficacy measure.** The *Sources of Middle School Mathematics Self-Efficacy Scale* (Usher & Pajares, 2009) was used to assess student math self-efficacy. The scale has a Cronbach's alpha of .95 and measured four constructs of (a) mastery experience (past successes and failures); (b) vicarious experience (experience by watching others); (c) social persuasions (by peers and others); and (d) psychological state. Items are written as first-person statements, and students are asked to rate how true or false each statement was for them on a scale from 1 (definitely false) to 6 (definitely true).

## **Procedures**

**Assent and consent procedures.** Once IRB approval was given, the primary investigator met with the school director who recommended possible teachers to participate. The primary investigator met with the individual teachers, explained all aspects of the study, and answered questions. Once they signed consent, teachers recommended possible students to participate. The primary investigator then met with students to explain all aspects of study and parental consent forms were sent home. After parents signed the consent forms, students were individually read aloud the student assent form and asked to sign.

**Pre- and post-assessment procedures.** The primary investigator collected demographic data on students and teachers prior to pretesting to ensure that all students meet criteria for inclusion. The data on the students were collected from the Individualized Education Plans for each student (see appendices A and B).

First, students were administered the MBSP by the classroom teacher to determine if the students had the necessary computation skills with a calculator. Next, students were individually administered the *KeyMath3-R* (Connolly, 1998) numeration and algebra subtests and self-efficacy survey by the primary investigator. Third, students took the fraction pretest on one day and on the next day completed the algebra pretest. Fourth, students received instruction on the fraction graphic organizer for five days. Fifth, students completed the fraction and algebra posttest 1 across two consecutive days. Sixth, students received instruction on the algebra graphic organizer for five days. Seventh, students took the fraction and algebra posttest 2 across two consecutive days. Eighth, one week later, students were given both the fraction and algebra maintenance tests each on a separate day. Ninth, students were individually administered the *KeyMath3-R* and self-efficacy survey as the posttest by the primary investigator. Finally, students and teachers completed the TARF-R and individual interviews. For each researcher



administered assessment, the researcher administered the test and students were not given any assistance.

**Graphic organizer intervention procedures.** Graphic organizer instruction took place classwide during the 60-minute math class three times per week for 30-45 minutes. If researchers observed fidelity below 90%, then booster training sessions were conducted before lessons resumed. The primary investigator met with the teacher before each lesson and reviewed the fidelity checklist and the steps to completing the graphic organizer. Lessons were scripted to demonstrate examples of proper student questioning and teacher use of consistent math language. Scripted use was optional. Lessons were based on the SRSD framework.

The lessons assisted teachers in leading students through six stages (see Appendix C for sample lessons). All lessons required teachers to (a) develop the background knowledge; (b) discuss the graphic organizer; (c) memorize it; and (d) lesson wrap up. In the development of the background knowledge sections, teachers reviewed vocabulary or pre-requisite math skills. For the fraction lessons students reviewed the vocabulary words “fraction,” “numerator,” “denominator,” and “equivalent fractions.” In addition, students reviewed how to draw fractions and create equivalent fractions visually. For the algebra lessons students learned the vocabulary words “variable,” “inverse operations,” and “equation.” Students practiced using drawings to represent equations. In the discussion portion, the teacher showed the students a completed graphic organizer and students discussed it. The teacher asked students “What do they notice about the graphic organizer?” and “What are the benefits to using it?” In the memorization section, students practiced memorizing the skills and vocabulary that were used in that particular lesson. In the wrap-up section, the teacher summarized the lesson with a discussion of what they had learned and what students were to learn the next instructional day. After the first lesson,

teachers modeled the steps to completing the graphic organizer and in subsequent lessons the students practiced completing the graphic organizer with support and then independently.

Specific math steps to completing each graphic organizer are described below.

***Fraction graphic organizer.*** The teacher modeled the steps that students used to solve the fraction problems. First, the teacher provided a fraction computation problem in the numeric box such as  $\frac{1}{2} + \frac{1}{3}$ . Second, the teacher drew each fraction in the visual box using the denominator to decide how many pieces to divide the rectangle in and the numerator to determine how many of those pieces to shade in (e.g., in the fraction  $\frac{1}{3}$  the rectangle is divided into three pieces with one part shaded in). Third, the teacher explained that due to different denominators the two fractions could not be added or subtracted so a common denominator must be found. Fourth, the teacher used the two fraction denominators to decide how many pieces to divide each fraction into using horizontal lines (e.g., in the fractions  $\frac{1}{2}$  and  $\frac{1}{3}$  the teacher would divide the rectangles into thirds and halves respectively). Fifth, the teacher demonstrated that each rectangle now has the same number of pieces and a common denominator of six. Sixth, the teacher used the new shaded portions to determine the new equivalent fractions (e.g.,  $\frac{3}{6}$  and  $\frac{2}{6}$  for  $\frac{1}{2}$  and  $\frac{1}{3}$  respectively). Seventh, the teacher wrote these new fractions in both the numeric and visual boxes. Eighth, the teacher demonstrated adding or subtracting the numerators and leaving the denominator the same with a final answer of  $\frac{5}{6}$ . Finally, in the steps box, the teacher reviewed the steps to solve the problem and wrote: (a) I drew each fraction, (b) I divided my first rectangle into thirds, (c) I divided my second rectangle into halves, (d) I created equivalent fractions, (e) I added the numerators together, and (f) I kept the same denominator. See Appendix C for a completed graphic organizer for fractions. To provide scaffolding, students were provided with sentence stems with key vocabulary as a word bank and

students had to fill in the words “fractions,” “divided,” “thirds,” halves,” “added,” and denominator.”

***Algebra graphic organizer.*** First, in the numeric box the teacher provided an equation to solve such as  $2x + 5 = 15$ . Second, the teacher drew the equation in the visuals box using longer rectangles to represent  $2x$  and smaller squares for five and 15. Third, the teacher showed students to use inverse operations to isolate the  $2x$  (e.g., instead of adding 5, subtract 5 from both sides). Students were shown the inverse operation both numerically and visually in the respective boxes. Fourth, both numerically and visually, the teacher demonstrated using the inverse operation to isolate  $x$  by itself (e.g., instead of multiplying by two, divide by two on both sides of the equation). Finally, in the steps box, the teacher reviewed the steps and write: (a) I drew my equations, (b) I subtracted five from both sides, (c) I brought down  $2x$  and subtracted five from 15 to get 10, (d) I divided both sides by two, and (e) I divided ten by two to get two. To provide scaffolding, students were provided with sentence stems with key vocabulary as a word bank and students had to fill in the words “equations,” “subtracted,” “divided,” as well as the numbers and variables. See Appendix C for a sample algebra graphic organizer.

### **Experimental Design and Analysis**

A one-group nonequivalent dependent variables design (Shadish, Cook, & Campbell, 2002) with multiple measures in multiple waves was used to assess the effects of the graphic organizer on the math skills of the students. This design involves a single group of students tested on two scales that are conceptually similar, but only one of which is expected to change because of the treatment. For this experiment, the fraction measure was expected to change, while the algebra measure was expected to remain the same until algebra instruction is provided. When multiple repeated measures are used in conjunction with this design, and the patterns of

achievement are predicted, most plausible threats to internal validity can be ruled out (Bottge, Rueda, & Skivington, 2006). To answer research questions one, two, and five a repeated measures ANOVA was conducted to determine significant differences between each time point. For fractions and algebra wave one was the pretest, wave two was the measure after fraction instruction, wave three was the measure after all instruction had been completed, and wave four was the maintenance measure. For the standardized measure and the self-efficacy wave one was the pretest and wave two was the posttest. Two one-way analyses of variance (ANOVAs) with repeated measures (waves 1, 2, 3, 4) were conducted on the fraction and algebra quizzes and one-way ANOVAs with repeated measures (waves 1 and 4) were conducted on the KeyMath3-R subtests and the self-efficacy survey. Research questions three and four were answered by reporting means and standard deviations. The student interviews were analyzed descriptively.

## **Results**

### **Researcher Developed Measures, and KeyMath**

Table 5 shows the means standard deviations for all measures. There was a statistically significant effect of time on fraction quizzes  $F(1.701, 15.309) = 7.770, p < .05, \eta^2 = .46$ , and on algebra quizzes and  $F(1.617, 14.555) = 9.718, p < .05, \eta^2 = .52$ . Post hoc tests using the Bonferroni correction revealed that fractions quizzes showed statistically significant differences between wave 2 and wave 3 ( $p=.031$ ), and between wave 2 and wave 4 (maintenance) ( $p=.011$ ). No significant differences between wave 1 and wave 2 ( $p=.166$ ) or between wave 2 and wave 3 ( $p=.094$ ) or between wave 2 and wave 4 ( $p=1.00$ ) or between wave 3 and wave 4 ( $p=1.00$ ) were found. Comparisons for algebra quizzes showed significantly higher achievement for wave 3 compared to wave 2 (pre and post instruction) ( $p = .003$ ), and for wave 2 compared to wave 4 (maintenance) ( $p = .046$ ). No significant differences were found between test waves 1

and 2 ( $p=1.00$ ), between waves 3 and 4 ( $p=.656$ ) or between waves 1 and 4 ( $p=.121$ ). It should be noted that on the fraction pretests students treated the fractions as whole numbers and added the numerators and denominators. After fraction instruction, the majority of students still treated the fractions as whole numbers, while some used the graphic organizer instruction strategy. On the KeyMath3-R subtests, results indicated no significant differences in achievement in numeration,  $F(1,10) = 1, p = .09$ , or in algebra,  $F(1,10) = .102, p = .76$ .

### **Fidelity**

On adherence and quality of instruction, 14 (43%) of the lessons were observed by the primary investigator and six (43%) of those lessons were observed by a second observer. Inter-observer agreement was calculated by taking the number of agreements and dividing by the number of agreements plus the number of agreements and disagreements. Inter-observer agreement was calculated at 98%. The mean percentage for steps completed across the intervention for teacher A was 59%, for teacher B was 86%, and for teachers C and D was 95%. The quality of instruction for teachers A and B was low with teachers making multiple math errors, forgetting steps and partially using the math vocabulary. Anecdotally, teachers A and B struggled to create properly the equivalent fractions correctly despite multiple practice with the primary investigator. The quality of instruction for teachers C and D was high with the teacher consistently using the math vocabulary, calling on a variety of students, and performing the math operations correctly.

For student engagement, 14 (47%) of the lessons were observed and six (43%) of those were observed by a second observer. Inter-observer agreement was calculated at 95%. The mean percentage of intervals for student engagement for teacher A was 52%, for teacher B was 46%, and for teachers C and D was 75%. It should be noted that student engagement was low

for teacher C, who taught the first two lessons, but after teacher D began instruction student engagement was near 100% for four out of the five students.

### **Social Validity**

Student results on the TARF-R indicated that they felt they (a) were clear about the procedures of the study ( $M=5.45$ ); (b) found the graphic organizer acceptable ( $M=6.00$ ); (c) found the graphic organizer helped them want to participate in math class ( $M=5.63$ ); (d) were willing to use it in the future ( $M=5.27$ ); (e) found it reasonable ( $M=5.91$ ); (f) were confident it was effective ( $M=6.00$ ); and (g) overall liked the procedures ( $M=5.00$ ). Students indicated that they were neutral on whether or not there were disadvantages to using the graphic organizer ( $M=4.00$ ) and whether or not other students liked using the graphic organizer ( $M=4.36$ ). Teacher results on the TARF-R indicated (a) they were clear about the study procedures ( $M=5.67$ ); (b) found the graphic organizer acceptable ( $M=5.67$ ); (c) found it reasonable ( $M=5.00$ ); (d) felt there were some disadvantages ( $M=5.00$ ); (e) felt much time would be needed to implement instruction ( $M=5.33$ ); (f) were willing to work with other teachers on the graphic organizer ( $M=5.33$ ); (g) thought some undesirable side effects were likely ( $M=5.00$ ); and (f) would be willing to change their class routine ( $M=5.67$ ). Teachers indicated that they were neutral to disagreeing with (a) the likelihood the graphic organizer will make permanent improvements ( $M=2.67$ ); (b) their confidence level at how effective the instruction was ( $M=3.00$ ); (c) their students had serious problems in math ( $M=3.33$ ); (d) the instruction would disrupt their class ( $M=4.67$ ); (e) the graphic organizer was effective for them ( $M=3.67$ ); (f) affordability of the graphic organizer ( $M=3.00$ ); (g) liking the procedures ( $M=3.00$ ); (h) felt student would feel no discomfort ( $M=1.67$ ); (i) students' math abilities are not severe ( $M=4.67$ ); and (j) how well it fits into their curriculum ( $M=4.67$ ).

On the student interviews, students were overall positive about the graphic organizer instruction. Students indicated that they “liked using the visuals,” and it helped guide them and improve their math.” Students in particular liked using the drawings because it helped them to “see it better.” The majority of students (73%) indicated that the visual square was their favorite. Some reasons students liked the visual square better were because “I like drawing,” “it helped me to see it better,” and “I’m a visual learner.” One student indicated he or she did not like any of the squares because “I do not like math.” Another student liked the numeric square because “they could understand it better.” One student liked the steps square the best because “if you do your problem at the same time as the steps, it helps.” Four students indicated that the visual square helped them the most, three thought the numeric square helped them the most, two felt the steps square helped them the most, and two students indicated that both the numeric square and visual square together helped them the most. Reasons students felt the visual square helped them the most included: “because I like to draw,” “because I got to see how to work it out,” and “it’s just fun to draw the boxes.” Reasons some thought the numeric square helped them included: “I could understand it more,” and “because I could just solve the problem.” Two students felt the steps square helped them most because “it was easier,” and “when I see an example with steps, it’s the same thing with different numbers.” Two felt that both helped them the most because “it guided them more,” and “helped them fully understand the problem.” The majority of students (64%) felt that algebra was harder to learn, while two students (18%) felt that were fractions were harder. One student felt that “they both go together.” Seven students (64%) felt that they still needed to use all three squares of the graphic organizer while four students (36%) felt that they did not. Reasons students felt they still needed all three squares included: “if I just use one I won’t fully understand it and can answer it completely,” “because it’s difficult without all three,”

and “because you could solve the problem better.” Reasons students felt they did not need all three squares included: “because the steps aren’t necessary anymore,” and “because I learned how do it easier.” There were a variety of answers from students when asked how they felt at the beginning of the math lesson. One student stated they “did not want to do this,” and two other students stated they were “mad because they were in the middle of something else.” Four students were nervous or did not want to do this, “but it changed as the lesson went on.” Four students felt good or wanted to do this because “they were going to get math in today.”

Teachers A, B and D completed the teacher interviews. One teacher indicated that “I liked the algebra one better. Didn’t like fractions when I was in school, but did see getting the common denominator. The steps were also confusing,” Another indicated that they liked “Looking at the fractions visually, Kids don’t know their multiplication, but they could count the squares.” Teacher D liked how “It provides the students with a visual representation of how they obtained their answer.” All three teachers thought the visual helped their students the most because “Because when using words with these kids, comprehension is lower so seeing it helped them,” and “because they could see it.” On the easiest and hardest part to teach, one teacher stated “Numerical was easiest because I had prior knowledge of it. Visual was the hardest. I got confused so they got confused,” and the other teacher stated “Easiest- Visual, Hardest was the steps, because when you write out what’s in your head it’s a little difficult.” The third teacher did not answer this question. Teachers found algebra easier to teach because “fractions were harder to write between the squares,” and “Add/ subtract then multiply/divide was easy to remember.” Two teachers thought that it was necessary to teach all sections of the graphic organizer because “because you do have to review what you did,” and “because it is designed to help students visualize and remember what and how they were taught, in order to solve these



problems.” One teacher thought there was a difference between fractions and algebra because “Algebra is easier to teach. They saw  $x$  as multiplying, but knew the signs for fractions better,” and another stated “It seemed different using the graphic organizer. I think they understood fractions better,” while the third teacher thought “No difference when using the graphic organizer.” One teacher thought, “partitioning the boxes was difficult and moving the boxes in algebra was too.” On the scripted lessons teachers felt “Scripted lessons were harder to do due to behavior,” “They’re easy to follow, but when I lost my place, it was hard to get back,” and “It helped me, help them with familiarizing them with mathematical terms consistently, etc.”

Improvements teachers suggested to the instruction included: “An example at the beginning for what it(the graphic organizer) should look like. Have a good example on a chart for all week. Definitions on charts as well,” and “Add more lessons.” Suggestions for improvements to the lesson plans included: “Everything together before working the problem. Put everything from the lesson at the start and optional script at the end,” “Have different colors for prompts so it’s easier when you get lost,” and “Show students how to solve problems with and without the graphic organizer.”

### Self-efficacy

On the middle school mathematics self-efficacy survey, results indicated no significant differences for scores on mastery experience,  $F(1,10) = .025, p = .88$  vicarious experience,  $F(1,10) = 1.739, p = .22$  social persuasions,  $F(1,10) = .069, p = .80$  or psychological state,  $F(1,10) = .069, p = .80$ .

**Table 5**  
*Means and Standard Deviations by Measure and Test Wave*

Measure	Test Wave			
	1	2	3	4
Fraction Quiz				
M	19.30	36.50	43.80*	38.00*

SD	7.30	24.18	24.69	17.56
Algebra Quiz				
M	7.50	0	57.20*	30.50*
SD	23.72	0	34.67	28.20
KeyMath Numeration				
M	4.82	--	--	5.09
SD	2.27	--	--	2.07
KeyMath Algebra				
M	4.45	--	--	4.36
SD	1.97	--	--	2.01
Self-Efficacy ME				
M	3.85	--	--	3.89
SD	1.15	--	--	
Self-Efficacy VE				
M	3.79	--	--	4.28
SD	1.09	--	--	1.40
Self-Efficacy SP				
M	3.83	--	--	4.53
SD	1.23	--	--	1.39
Self-Efficacy PS				
M	3.25	--	--	3.14
SD	1.52	--	--	1.57

*Note.* -- indicates that the measure was not given at that time point; ME= Mastery Experience; VE= Vicarious Experience; SP=Social Persuasions; PS=Psychological State

## Discussion

The primary purpose of this study was to test the effects of graphic organizer instruction using an SRSD framework (Cuenco-Carlin et al., 2016) on fraction and algebra performance for both students and teachers in an AES. With regard to the first research question, students did significantly improve their ability to solve both fraction computation and two-step algebra equations indicating that the graphic organizer instruction did improve their math performance. It is encouraging that the students maintained the skills they were taught. However, fraction

scores were much lower than algebra scores. Despite graphic organizer instruction, the majority of students continued to treat fractions as whole numbers as most students with LD tend to do (Woodward, Baxter, & Robinson, 1999). However, on the algebra pretests, students did not have any prior knowledge on how to solve two step equations. In order to facilitate proper fraction instruction it is necessary to “unteach” student misconceptions about fractions (Woodward et al., 1999) before reteaching fractions. With algebra, no “unteaching” was necessary, which could account for why algebra scores were higher than fraction scores. In addition, social validity results indicated that two of the teachers were more comfortable with algebra, which could account for the higher scores. Furthermore, two teachers had lower fidelity, which could be due to the low amount of professional development. Overall, graphic organizer instruction shows promise as an effective intervention for students with EBD.

With regard to the second research question, students did not significantly improve their performance on the KeyMath3-R subtests (Connolly, 1996). The lack of significance with the KeyMath3-R could have been because it was a distal measure. Considering the KeyMath3-R test measures a wide range of skills (e.g., whole-number operations, decimals) in addition to fraction computation, it is not entirely surprising that the gains that students made on the researcher-developed measures did not show up on the standardized measure. It also is difficult to get a significant change on a standardized measure within a short time period of intervention (Bottge, Rueda, Grant, Stephens, & Laroque, 2010).

With regard to the third research question related to fidelity, the quality of instruction and student engagement for two of the teachers (A and B) was low, while for one teacher (D) it was high. This could explain why the students did not meet mastery. If the students received improper instruction and were not engaged, then they may have decided to use what they had

been taught previously (i.e., treat fractions as whole numbers). In addition, when interviewed, teachers A and B indicated that they were not as comfortable with fractions as algebra, which could also account for why the algebra scores were higher. In general, researchers have struggled to find ways to improve fraction knowledge for teachers who struggle with fractions instruction (Jayanthi, Gersten, Taylor, Smolkowski, & Dimino, 2017.) Jayanthi et al. (2017) suggested that a fraction professional development program that differentiates instruction for teachers with strong math knowledge versus those who have far less initial understanding of fractions might be needed. The fidelity results of this study supports this idea because teachers A and B were not comfortable with fractions and required more intensive instruction while teacher D required little support after the initial professional development training.

Differentiated instruction during the initial training may have led to higher fidelity scores. In addition, adding a content measure following professional development may have helped increase scores.

Teachers were provided scripted lessons to help them with adherence to the instruction. Teachers indicated that they felt the scripted lessons helped them use consistent vocabulary. Increasingly, researchers acknowledge the importance of concise math language from grade level to grade level when providing instruction to students (Hughes, Powell, & Stevens, 2016). Although teachers indicated in the teacher interviews that the scripts could be difficult to follow when they lost their place, using scripts allowed the teachers to use consistent math language.

Student engagement was low for teachers A and B, which could also explain why the students did not master fractions or algebra. Students with EBD tend to struggle with on-task behavior (Haydon et al., 2012), and with motivation (Wehby, Falk, Barton-Arwood, Lane, & Cooley, 2003). In AES, some researchers have been required to add a behavior component to

their intervention in order to enhance academics (e.g., Bowmann, Perrot, Greenwood, & Tapia, 2007). Students in AES often exhibit low instructional motivation (Lehr et al., 2009) and may require behavior strategies along with academic interventions in order to succeed academically. Bowmann et al. (2007) offered a reward to their students when they effectively self-managed their behavior and students in this study may have benefited from a reward for high student engagement.

With regard to the fourth research question, overall students and teachers found the graphic organizer instruction to be socially acceptable. It is important to note that students who did not like math did not change their opinions after receiving the graphic organizer instruction. These students were also the students that continued to have low engagement. The low engagement of students who did not like math helps explain their low scores. It is interesting that some students perceived fractions to be easier than algebra. This could be because their misconceptions about fractions were not directly addressed. These students could have thought fractions were easier than algebra due to treating them as whole numbers (Woodward et al., 1999). In addition, it should be noted that students never saw their scores on pretests or posttests. When students self-monitor their math performance, they tend to do better academically (Gersten et al., 2008; Shimabukuro, Prater, Jenkins, & Edelen-Smith, 1999). The student perceptions may have been different if they had seen their math scores.

With regard to the fifth research question, student self-efficacy scores did not significantly improve because of graphic organizer instruction. Students remained neutral on all four constructs. However, the power was low on all four constructs ( $R=.223-.57$ ). If the study had used a larger number of students, then the results may have been different. However, it is

important to examine the relationship between student math performance and self-efficacy more closely (Hughes & Riccomini, 2011) with a larger number of students.

### **Limitations and Future Directions**

There are several limitations that should be taken into consideration when interpreting the present study's results. First, the power using the ANOVA's on the fraction measures was lower (.87) due to low number of students and a lower effect size. Since the difference between fraction scores was not as large as the algebra scores, the power was lower. Therefore, it is more difficult to know whether or not there was a statistical difference between time points on the fraction measures. Future studies should attempt to replicate the results with a larger number of students.

Second, this study used no control group. Without a control group, it is difficult to tell whether the graphic organizer instruction is a more effective intervention than typical classroom instruction. However, these students had a history of math difficulties and were attending an AES indicating that typical classroom instruction had not been effective for them (Lehr et al., 2009). In addition, the lack of a control group may lead to inflated effect sizes (Borenstein, Hedges, Higgins, & Rothstein, 2011). Future studies should examine the effects of the graphic organizer instruction using a control group to compare it to typical instruction and calculate an effect size, which is not inflated.

Third, the students did not meet mastery level (80% or higher) for the mean percentage scores for fractions ( $M=43.80$ ) or algebra ( $M=57.20$ ). This could be because the students only received five lessons on each type of math problem. Two studies on algebra for students with LD (Witzel, 2005; Witzel et al., 2003) spent at least one month on algebra equations to help students with mastery. Since students in AES are several grade levels below their peers (Ruzzi

& Craemer, 2006), they may need more than five lessons on fractions and algebra to demonstrate mastery. Future researcher should examine the duration and length it takes for students with EBD in AES to master fractions and algebra. Fourth, for two teachers the fidelity, quality of instruction, and student engagement were low. This could explain some of the variation of the fraction and algebra scores in the group. Future researchers should examine ways to improve teacher fidelity particularly in fraction instruction as well as ways to improve student engagement for students with EBD in AES.

## **Conclusion**

Students with EBD in AES really struggle in math (Schwab et al., 2016) and require supports to improve their math performance. Graphic organizer instruction led to some promising results with this population. However, other factors such as teacher math knowledge, fidelity, student engagement, and self-efficacy have an impact on their math performance. This study examined each of these, but with a limited number of participants, it was difficult to see some statistical differences. The findings from this study suggest that initial graphic organizer instruction can improve fraction and algebra performance, but more time may be needed for these students to reach mastery. Examining these factors in relation to the graphic organizer instruction may lead to improved math outcomes and help students with EBD in AES, which in turn, will help them graduate and obtain better employment. This study is a good first step in examining higher math skills for this population. With more research on fractions and algebra instruction, students with EBD in AES may obtain more positive math results and return to their regular education schools.

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**APPENDICES****Appendix A: Student Demographic Form**

Name (First, middle, Last): \_\_\_\_\_

Date of Birth: \_\_\_\_\_

Gender (Circle one): Male or Female

Grade: \_\_\_\_\_

Race (Circle one): White, Hispanic or Latino, Black or African American, Native American or American Indian, Asian/Pacific Islander.

Primary Disability (Circle all that apply): EBD, LD, OHI, LI

Secondary Disability (Circle all that apply): EBD, LD, OHI, LI

IQ: \_\_\_\_\_

Writing Score: \_\_\_\_\_

Working Memory: \_\_\_\_\_

Free Lunch (yes or no) \_\_\_\_\_

Math Goals in IEP (yes or no): \_\_\_\_\_

Write down Math goals and any related goals:

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**Appendix B: Teacher Demographic Form**

Name (First middle, Last):

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Date of birth: \_\_\_\_\_

Gender (Circle one): Male or Female

Race (Circle one): White, Hispanic or Latino, Black or African American, Native American or American Indian, Asian / Pacific Islander.

Highest degree earned (Circle one): Bachelor's, Master's, Doctoral

Number of Years teaching: \_\_\_\_\_

Number of Years teaching in this type of setting: \_\_\_\_\_

Certification (Circle all that apply): General Education Math Teacher, Special Education Teacher of Students with mild disabilities, Special Education Teacher, General or Cross-categorical, Elementary Education Teacher, Secondary Education Teacher; subject area(s):

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Other (Please specify): \_\_\_\_\_

## Appendix C: Sample Fraction Lesson plans

### Lesson One: Developing Background Knowledge and Introducing the Graphic Organizer

*(this lesson may take 1 day)*

#### Lesson Overview:

1. The teacher will introduce students to the vocabulary.
2. The teacher will introduce the students to the graphic organizer.
3. Students will discuss thoughts about the graphic organizer and fractions.
4. Students will be familiar with the following terms: **fraction, numerator, denominator, equivalent fractions.**

(Prepare by drawing H on the chart paper before class begins for discussion)

#### Set the Context for Learning:

**Explain to students that they are going to learn a new strategy that will help them learn to add and subtract fractions problems. Explain that they are going to learn to solve fraction problems with drawings and numbers.**

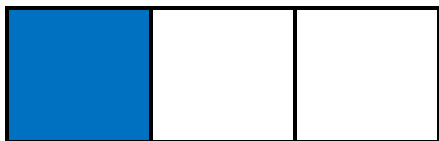
#### Step 1: Develop the Background Knowledge

**Explain that a fraction is a part of a whole. Tell the students we are going to look at a few examples of what fractions are.**

**Ask students where they see fractions in real life?**

Students: *dividing a pizza into slices or dividing candy into equal pieces.*

*Draw the following fraction on the board.*



**Explain that the box is divided into three total pieces and that one of the parts is shaded so that means 1 part out of 3 are shaded so that would be:**

$$\frac{1}{3}$$

**Explain that the top part is the numerator and the bottom part or whole is the denominator.**

*Label each part of the fraction here*

$$\frac{1 \text{ Numerator}}{3 \text{ Denominator}}$$

*Draw the next fraction on the board*

A.



**Ask students:**

**How many total pieces are there?**

*Students: 5 total pieces*

**What portion is shaded?**

*Students: 3 are shaded*

**How do we write this as a fraction?**

*Students: 3 on top and 5 on the bottom*

**Which is the numerator**

*Students: 3 is the numerator*

**Which is the denominator?**

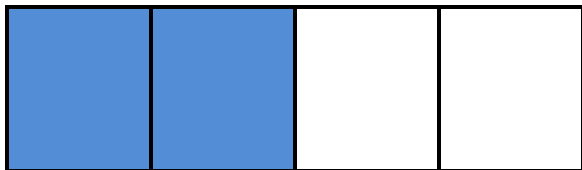
*Students: 5 is the denominator*

$$\frac{3 \text{ Numerator}}{5 \text{ Denominator}}$$

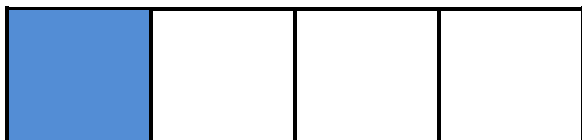
*Draw B and C on the board*

**You work the next two problems on your own.**

B.



C.



**Ask students:**

**How many total pieces are there?**

*Students: 4 total pieces*

**What portion is shaded?**

*Students: 2 are shaded*

**How do we write this as a fraction?**

*Students: 2 on top and 4 on the bottom*

**Which is the numerator**

*Students: 2 is the numerator*

**Which is the denominator?**

Students: *4 is the denominator*

Answer:  $\frac{2 \text{ Numerator}}{4 \text{ Denominator}}$

**How many total pieces are there?**

Students: *4 total pieces*

**What portion is shaded?**

Students: *1 is shaded*

**How do we write this as a fraction?**

Students: *1 on top and 4 on the bottom*

**Which is the numerator**

Students: *1 is the numerator*

**Which is the denominator?**

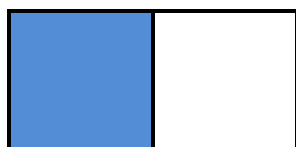
Students: *4 is the denominator*

Answer:  $\frac{1 \text{ Numerator}}{4 \text{ Denominator}}$

**Tell students next we need to review what equivalent fractions are. Equivalent fractions are equal fractions. That means that they are the same.**

Draw the following two fractions on the board.

D.



E.





**What is the name of this fraction?** (point to D)

Students: *one half*

$$1/2$$

**What is the name of this fraction?** (point to E)

Students: *three sixths*

$$3/6$$

**Ask students what do you notice about these two fractions?**

*Point to D and E*

Students: **They the same size?**

**What do you notice about the shaded areas?**

Students: The shaded areas are the same size.

**That right! Although we have just cut the fractions into a different number of pieces the shaded portions are still the same. Therefore, the fractions are equivalent.**

*Write problem F on the board*

**We need to find out if these two fractions are equivalent. First, I am going to draw  $1/2$  and  $2/4$  in the squares provided and see if they are equal.**

*Draw  $1/2$  and  $2/4$  in the appropriate boxes*

F.

$$1/2 \text{ and } 2/4$$

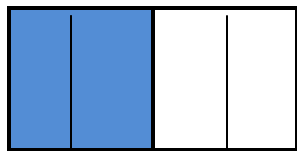
*Answers:*







$$\frac{1}{2}$$



$$\frac{2}{4}$$

**Are these equivalent fractions and why or why not?**

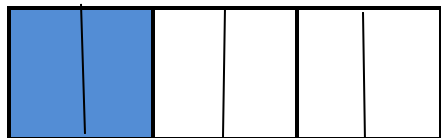
Students: *Yes, they are equivalent because the same portion is still shaded when you divide the box into 4 equal pieces.*

**Excellent! The two fractions are equivalent or equal because the same portion is still shaded when the same size box shaded into four pieces.**

*Draw G on the board.*

G.

$$\frac{1}{3} \text{ and } \frac{2}{6}$$



**Are these two fractions equivalent?**

Students: *yes*

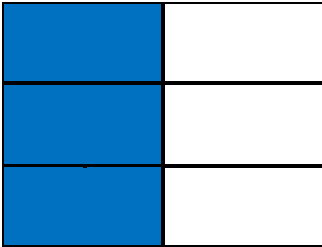
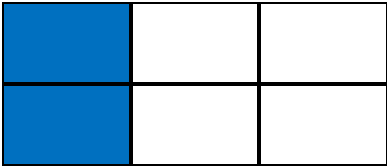
**That's right! The same portion is shaded so these are equivalent fractions.**

**Step 2: Discuss the Strategy**

Tell students for the next two weeks we are going to learn to solve fraction problems in a different way using some of what we've learned today.

*Show them H on the board*

H.

Numerical	Visual	Steps
$\frac{1}{2} + \frac{1}{3} =$ $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$	 $\frac{1}{2} = \frac{3}{6}$  $\frac{1}{3} = \frac{2}{6}$	<ol style="list-style-type: none"> <li>1. I drew my fractions.</li> <li>2. I divided my first square into thirds</li> <li>3. I divided my second fraction square into halves</li> <li>4. I created equivalent fractions.</li> <li>5. I added the numerators together.</li> <li>6. I kept the same denominator.</li> </ol>

Ask students to discuss in pairs what they notice about the graphic organizer.

Ask the pairs to share out what they notice.

Discuss the ways this graphic organizer may benefit them such as: It will help them better understand fractions. It will help them organize the different ways of looking at a fraction. It will help them better remember the steps to adding and subtracting fractions. Emphasize that there are different denominators so you cannot simply add the fractions without a common denominator.

**Step 3: Practice Memorizing the vocabulary**

**Turn and talk your partner and review the definition for fraction, numerator, denominator, and equivalent fractions.**

**Step 4: Wrap-up:**

**Congratulate the students for doing a good job today! Tell the students today we went over some vocabulary and fraction concepts and introduced the graphic organizer. Tell them next time we will work together through the graphic organizer.**

## Lesson Two: Modeling each Step of the Graphic Organizer

*(this lesson may take 1 day)*

### Lesson Overview:

1. The teacher will model each step to completing the graphic organizer.
2. The teacher will model self-talk and think alouds.

### Set the Context for Learning:

**Remind students that this week they are learning a new way to solve adding and subtracting fraction problems. Previously, we learned and memorized vocabulary, how to draw fractions, and how to tell if two fractions are equivalent.**

### Step 1: Develop the Background Knowledge

**Let's review what we learned from yesterday.**

*Point to or draw A on the board.*



**$\frac{3}{4}$**

**We have the fraction  $\frac{3}{4}$ . What is a fraction?**

Students: *part of a whole number.*

**That's right! A fraction is a part of a whole number. The numerator is the part of the whole number. Which number is the numerator?**

Students: *the numerator is 3.*

**That's right! The numerator is top number which is three. Which number is the denominator?**

Students: *The denominator is 4.*

**Excellent! The denominator is 4 in this case so let's label these.**

*Label the numerator and denominator and have students do the same on their sheets*

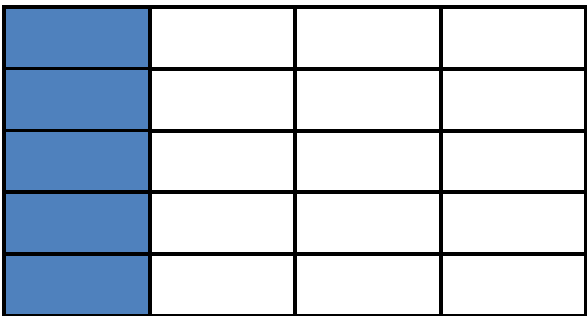
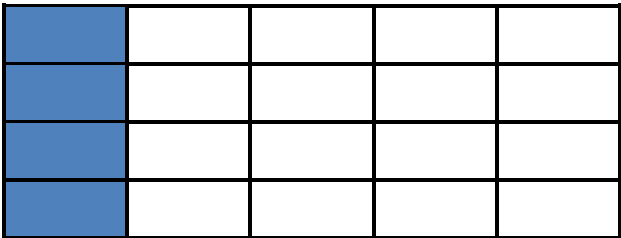
## $$\frac{3 \text{ Numerator}}{4 \text{ Denominator}}$$

### Step 2: Discuss the Graphic Organizer

*Show students the following completed graphic organizer (B)*

Remind them that they are going to learn how make one of these today by watching the teacher. Ask students to discuss in pairs what they notice about the graphic organizer and share with the class. (Guide students to see that this problem is subtraction. Allow them to make the connections between the picture and the drawing).

B.

Numerical	Visual	Steps
$\frac{1}{4} - \frac{1}{5} =$ $\frac{5}{20} - \frac{4}{20} =$ $\frac{1}{20}$	 $\frac{1}{4} = \frac{5}{20}$  $\frac{1}{5} = \frac{4}{20}$	<ol style="list-style-type: none"> <li>1. I drew my fractions.</li> <li>2. I divided my first square into fifths</li> <li>3. I divided my second square into fourths.</li> <li>4. I created equivalent fractions.</li> <li>5. I added the numerators together.</li> <li>6. I kept the same denominator.</li> </ol>

### Step 3: Modeling the Steps to Completing the Graphic Organizer

**Tell Students that they are now going to watch you work through an entire graphic organizer. Ask them to notice what you do for each step and the way you think about the problem. Start with the following graphic organizer.**

*Draw C on the board.*

**C.**

Numerical	Visual	Steps
$\frac{1}{3} + \frac{3}{5} =$		

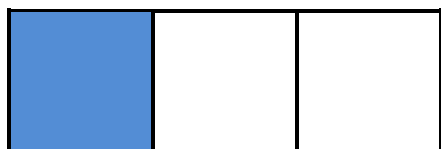
**“Alright, the first thing I have to is draw my fractions in the visuals box. My denominator is 3 so I will divide this square into 3 equal pieces and my numerator is one so I will shade in one of those pieces. In order to divide the square into 3 equal pieces I need to have two lines.**

*Draw two lines and label the fraction 1/3 in the visuals box.*

**Now next, I need to draw 3/5. My denominator is 5 so I will divide the square into 5 equal pieces and shade in 3 squares for my numerator. In order to divide the square into five equal pieces I need to have four line.**

*Draw four lines in the square and label the fraction 3/5 in the visuals box.*

Visual square should look like this.



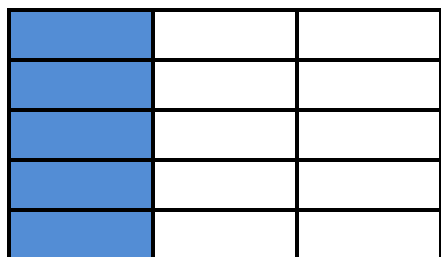
$\frac{1}{3}$



$\frac{3}{5}$

**“Now I ask myself if I can add these two fractions together. I can’t since they have different denominators. So first I need to find equivalent fractions so I will look at the denominator of my other fraction to determine how many pieces to divide  $\frac{1}{3}$  into. I see that the denominator is 5 so I need to divide my first square into fifths and I can do that horizontally or across.**

*Draw the four horizontal lines on the drawing as shown below*

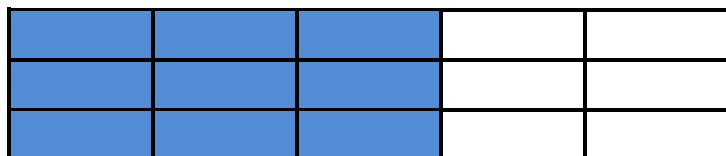


$\frac{1}{3}$

**So now I have divided my one third into five pieces and I now have a total of 15 total pieces. Now let’s divide my  $\frac{3}{5}$ . I look at my denominator here and I see that it is 3 so I will divide**

my second rectangle into thirds and see what happens. Once again I'm going to draw the lines horizontally to make it fit."

*Draw two horizontal lines on the drawing as shown below*



$\frac{3}{5}$

So now I have divided my second square into thirds and have 15 equal pieces so my common denominator is 15.

(Note Graphic Organizer should look likes this)

Numerical	Visual	Steps
$\frac{1}{3} + \frac{3}{5} =$	<div data-bbox="508 1113 940 1365"> </div> <p data-bbox="495 1402 537 1436"><math>\frac{1}{5}</math></p> <div data-bbox="508 1474 1227 1625"> </div> <p data-bbox="495 1663 537 1696"><math>\frac{3}{5}</math></p>	



Now lets write down 15 as my common denominator in the numerical square. My new fraction is 5/15 for the first one and 9/15 for the second fraction so lets write that down in the numerical square.”

*Write down 5/15 + 9/15 in the numerical square.*

(Graphic Organizer should look like this)

Numerical	Visual	Steps
$\frac{1}{3} + \frac{3}{5} =$ $5/15 + 9/15 =$	<div data-bbox="508 642 940 894"> </div> <p data-bbox="496 936 626 968"><math>1/3 = 5/15</math></p> <div data-bbox="508 1005 1227 1157"> </div> <p data-bbox="496 1199 618 1230"><math>3/5 = 9/15</math></p>	

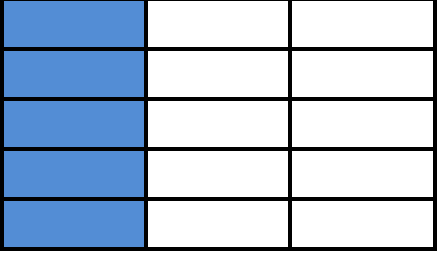
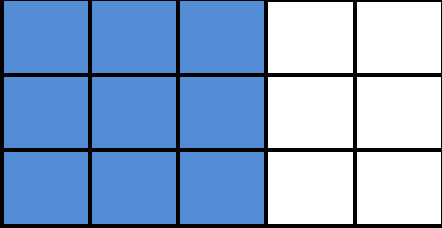
Now that I have my equivalent fractions I just need to add my numerators and keep the same denominator.  $5 + 9 = 14$  and the denominator is 15 so my final answer is 14/15.

*Write down 14/15 in the numerical square.*

And finally I need to list my steps in the third box. So first, I drew my fractions, next I divided my thirds box into fifths. Third, I divided my fifths box into thirds. Fourth, I created my equivalent fractions. Fifth, I added my numerators and kept the same denominator.”

*List each step in the graphic organizer as you say it.*

(Final Graphic organizer should look like this)

Numerical	Visual	Steps
$\frac{1}{3} + \frac{3}{5} =$ $\frac{5}{15} + \frac{9}{15} =$ $\frac{14}{15}$	 $\frac{1}{3} = \frac{5}{15}$  $\frac{3}{5} = \frac{9}{15}$	<ol style="list-style-type: none"> <li>1. I drew my fractions.</li> <li>2. I divided my thirds box into fifths.</li> <li>3. I divided my fifths box into thirds.</li> <li>4. I created equivalent fractions.</li> <li>5. I added the numerators together. I kept the same denominator.</li> </ol>

Ask the students what did they notice as you worked the problem. Lead them toward describing the steps to add or subtract fractions.

**Step 4: Practice memorizing the steps to adding or subtracting fractions.**

Have the students work in pairs to say the steps to each other on how to add and subtract fractions or allow them to write them down on the worksheet.

**Step 5: Wrap-up**

Tell the students Great job today! Today you watched me work on the graphic organizer and were introduced to each step. Tell students that they practiced memorizing it today and tomorrow, you and I will work a problem together.

### **Lesson Three: Collaborative Modeling each Step of the Graphic Organizer**

*(this lesson may take 1 day)*

#### **Lesson Overview:**

1. The teacher and student will work together to complete the graphic organizer.
2. The students will be practice saying each step to complete.

#### **Set the Context for Learning:**

**Remind students that this week they are learning a new way to solve adding and subtracting fraction problems. Previously, we learned and memorized vocabulary, how to draw fractions, and how to tell if two fractions are equivalent, and you watched me make the graphic organizer.**

#### **What is a fraction?**

Students: *part of a whole number.*

**That's right! A fraction is a part of a whole number. The numerator is the part of the whole number. What is the numerator?**

Students: *the part that is shaded.*

**That's right! The numerator is the part. What is the denominator?**

Students: *The total number of pieces.*

**Correct! And what are equivalent fractions?**

Students: *Two fractions that are equal.*

#### **Step 1: Develop the Background Knowledge**

*Point to A (Let Students work on their own)*

*Draw the fraction  $\frac{2}{5}$  and label the numerator and denominator.*



Help students as necessary. Check student work together by asking students to share their drawings and labels.

### **Step 2: Discuss the strategy**

**What are the three steps to creating the graphic organizer.**

Students: *Make a visual or drawing, Write it down numerically. Write down the steps to solving.*

### **Step 3: Collaborative Modeling**

*Write problem B on the board*

B.

$$3/5 - 1/4 = ?$$

**Let's use our graphic organizer to solve this problem. First I need to write the problem down in our graphic organizer.**

*Write  $3/5 - 1/4$  in the numeric square*

**Which square I should put the problem in?**

Students: *The numeric square*

**That's right! We put the problem in the numerical square.**

**Now who can tell me the next step to solving the problem.**

Students: *You should next draw the fractions  $3/5$  and  $1/4$ .*

**Excellent! We draw our fractions. "So how many squares should I divide my first square into?"**

Students: *Four squares*

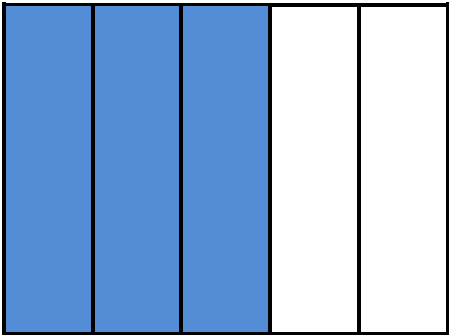
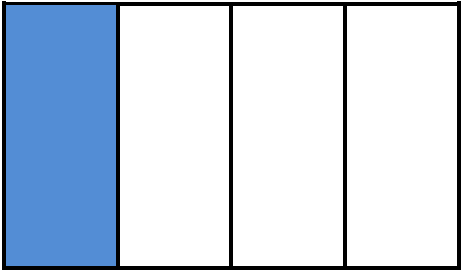
That's right! We look at the denominator to see how many horizontal lines to draw and in this case it's four. Now how many squares should I shade in?

Students: *3 of them*

Perfect! Yes the numerator is three in this case so we need to shade in three of the squares. Now who wants to come on the board and draw the fraction  $1/4$ .

*Call on one student to draw  $1/4$  on the chart or board.*

(Graphic organizer should look like this).

Numerical	Visual	Steps
$\frac{3}{5} - \frac{1}{4} =$	 <p><math>3/5 =</math></p>  <p><math>1/4 =</math></p>	<p>I drew my _____.</p> <p>I _____ my first fraction box into _____.</p> <p>I _____ my second fraction into _____.</p> <p>I created _____</p> <p>Fractions.</p> <p>I _____ the numerators and kept the same _____.</p>

What is the next step to solving the problem?

Students: *Divide the first fraction into fourths with three horizontal lines and the second fraction into fifths with four horizontal lines.*

*Draw three horizontal lines in the first square and four in the second*

(Graphic organizer should look like this)

Numerical	Visual	Steps
$\frac{3}{5} - \frac{1}{4} =$	<div data-bbox="454 556 917 871"> </div> <p data-bbox="454 903 519 945"><math>3/5 =</math></p> <div data-bbox="454 987 933 1291"> </div> <p data-bbox="454 1344 519 1386"><math>1/4 =</math></p>	<p data-bbox="1047 556 1559 619">I drew my _____.</p> <p data-bbox="1047 672 1567 840">I _____ my first fraction box into _____.</p> <p data-bbox="1047 892 1567 1060">I _____ my second fraction into _____.</p> <p data-bbox="1047 1113 1559 1165">I created _____</p> <p data-bbox="1047 1218 1234 1270">Fractions.</p> <p data-bbox="1136 1333 1542 1533">I _____ the numerators and kept the same _____.</p>

**Excellent! What is the next step to solving this problem?**

Students: *Write down your equivalent fractions.*

**Great! What is my first equivalent fraction?**

Students: *12/20*

**That's right! 12 is the numerator since it is the portion shaded and the total number of squares is now 20. What is the equivalent fraction for the second box.**

Students:  $5/20$ .

**Good job! Yes, 5 is the numerator since it is the portion shaded and the total number of squares is now 20 the same as the first one.**

*Write these fractions in the numeric and visual square*

**Who can tell me the next step?**

Students: *We subtract the numerator and keep the same denominator.*

**Excellent! Yes, we subtract our numerators  $12 - 5$ , which is 7 and keep the same denominator, which is 20. Our final answer is  $7/20$ .**

**So, for our first step we drew our ?**

Students: *Fractions*

**Then we**

Students: *divided*

**Our first fraction box into**

Students: *Fourths*

**Third we divided our second fraction into**

Students: *Fifths*

**Fourth, we created**

Students: *Equivalent*

**Fifth, we**

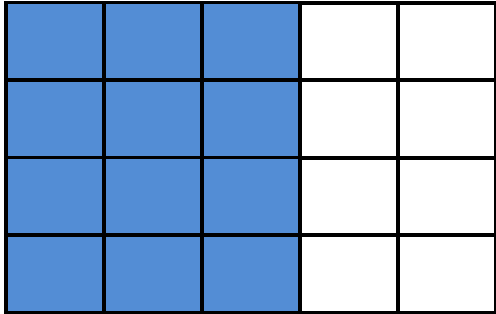
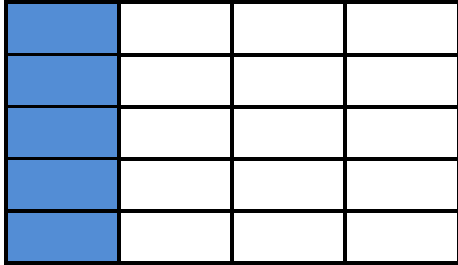
Students: *Subtracted*

**The numerators and kept the same**

Students: *denominator*

*Fill in the blanks with the appropriate words on the board*

(Complete graphic organizer is shown below)

Numerical	Visual	Steps
$\frac{3}{5} - \frac{1}{4} =$ $12/20 - 5/20 = 7/20$	 $3/5 = 12/20$  $1/4 = 5/20$	<ol style="list-style-type: none"> <li>1. I drew my fractions.</li> <li>2. I divided my first fraction box into fourths.</li> <li>3. I divided my second fraction into fifths.</li> <li>4. I created equivalent fractions.</li> <li>5. I subtracted the numerators together. I kept the same denominator.</li> </ol>

#### Step 4: Memorize the strategy (Give them five minutes before moving on)

Take down the graphic organizer so students have no hints and ask them to write them the steps to adding and subtracting fractions. Do not let them use notes!!!

#### Step 5: Wrap-up

Tell students great job today! Today we worked together at completing a graphic organizer and tomorrow students will work independently at completing the graphic organizer.



## **Lesson Four: Independent Practice for completing the Graphic Organizer or Guide Practice**

*(this lesson may take 1 day)*

### **Lesson Overview:**

1. The students will complete the graphic organizer independently.

### **Set the Context for Learning:**

**Remind students that this week they are learning a new way to solve adding and subtracting fraction problems. Previously, we worked together to complete the graphic organizer. Today, we will split into groups with some of you working independently to complete one and others working in pairs with the teacher.**

### **Step 1: Develop the Background Knowledge**

**What are the three steps to creating the graphic organizer.**

Students: *Make a visual or drawing, Write it down numerically. Write down the steps to solving.*

**Great and what are the steps to adding or subtracting the fractions**

Students: *Draw the fractions, Divide the first fraction box, Divide the second fraction box, Create equivalent fractions, add or subtract the numerators and keep the same denominator.*

### **Step 2: Discuss the strategy**

**Excellent! This graphic organizer is useful because it allows us to use visuals and remember the steps.**

### **Step 3: Guided Support or Independent Practice**

Depending on the memorization quizzes the previous day. Divide the students into two groups. Those who remembered all the steps to solving the problem will work independently on a practice problems.

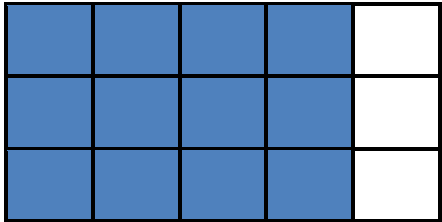
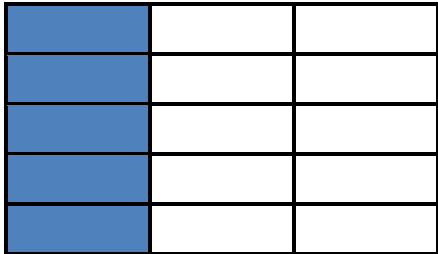
Group one is given the following problem:

A.  $\frac{4}{5} + \frac{1}{3} = ?$

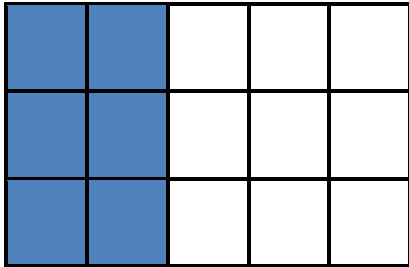
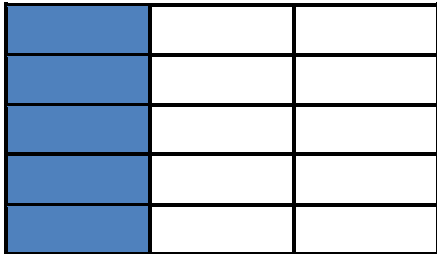
Group 2 works in pairs and with guided support to solve the following problem:

B.  $\frac{2}{5} + \frac{1}{3} = ?$

A.

Numerical	Visual	Steps
$\frac{4}{5} + \frac{1}{3} =$ $12/15 + 5/15 =$ $17/15$	 <p><math>4/5 = 12/15</math></p>  <p><math>1/3 = 5/15</math></p>	<ol style="list-style-type: none"> <li>1. I drew my fractions.</li> <li>2. I divided my first fraction box into thirds.</li> <li>3. I divided my second fraction into fifths.</li> <li>4. I created equivalent fractions.</li> <li>5. I subtracted the numerators together. I kept the same denominator.</li> </ol>

B.

Numerical	Visual	Steps
$\frac{2}{5} + \frac{1}{3} =$ $6/15 + 5/15 =$ $11/15$	 <p><math>2/5 = 6/15</math></p>  <p><math>1/3 = 5/15</math></p>	<ol style="list-style-type: none"> <li>1. I drew my fractions.</li> <li>2. I divided my first fraction box into thirds.</li> <li>3. I divided my second fraction into fifths.</li> <li>4. I created equivalent fractions.</li> <li>5. I subtracted the numerators together. I kept the same denominator.</li> </ol>

**Step 4: Memorize the steps:**

*Clear away all notes and ask students who struggled to write down the steps to completing the graphic organizer and to add and subtract and fractions.*

**Step 5: Wrap- up:**

**Tell students good job today! Today we filled out graphic organizers independently and tomorrow we will practice adding and subtracting fractions without graphic organizers.**

## Lesson Five: Fraction Computation Practice

*(this lesson may take 1 day)*

### Lesson Overview:

2. The students will independently complete three fraction problems.

### Set the Context for Learning:

Remind students that this week they are learning a new way to solve adding and subtracting fraction problems. They have been practicing completing graphic organizers and today they will just solve fraction problems.

### Step 1: Develop the Background Knowledge

**What are the three steps to creating the graphic organizer.**

Students: *Make a visual or drawing, Write it down numerically. Write down the steps to solving.*

### Great and what are the steps to adding or subtracting the fractions

Students: *Draw the fractions, Divide the first fraction box, Divide the second fraction box, Create equivalent fractions, add or subtract the numerators and keep the same denominator.*

### Step 2: Discuss the strategy

**Tell students to remember how they used the graphic organizer to solve organize their diagrams and now they will practice without them.**

### Step 3: Independent Practice

*Give the students the following three fraction problems and monitor them working.*

A.  $\frac{1}{4} + \frac{2}{5} =$

B.  $\frac{1}{3} + \frac{2}{4} =$

C.  $\frac{4}{5} - \frac{1}{2} =$

### Step 4: Wrap-Up

**Tell students Good job today! Tomorrow they will take the fraction computation posttest.**

## Appendix D: Sample Algebra Lesson Plan

### Lesson Six: Developing Background Knowledge and Introducing the Graphic Organizer

*(this lesson may take 1-2 days)*

#### Lesson Overview:

The teacher will introduce students to the vocabulary.  
 The teacher will introduce the students to the algebra graphic organizer.  
 Students will discuss thoughts about the graphic organizer and algebra.  
 Students will be familiar with the following terms: **variable, inverse operation, equation.**

#### Set the Context for Learning:

**Explain to students that they are going to learn a new strategy that will help them learn to solve algebra equations. Explain that they are going to learn to solve algebra equations with both drawings and numbers.**

#### Step 1: Develop the Background Knowledge

**Explain that a variable is a letter that stands for a number. Tell the students we are going to look at a few examples of what variables are.**

*Draw the following on the board.*

$$X - 2 = 6$$

**Explain that a variable is a letter that stands for a number. Explain that we do not know what the number is yet. So, let's write that down.**

*Write down the definition for variable- A letter that stands for a number*

**In this problem  $X - 2 = 6$ , what is our variable?**

Students: X

**That's right! X is our variable so let's circle the X on our paper.**

*Circle X on the board and have students do the same on their paper*

**Ask the students what number do we subtract from 2 to get 6.**

Students: 8

**Right in this case X stands for the number 8.**

**Tell students that you are going to look more closely at how we figured that out.**

**First we need to know that the opposite of subtraction is adding. The opposite of subtracting is adding. The opposite of multiplication is division and the opposite of multiplication is division.**

**What is the opposite of adding?**

Students: *Subtraction*

**That's right! And what is the opposite of Subtraction**

Students: *Adding*

**Correct! And what's the opposite of multiplication**

Students: *Division*

**Excellent! And the opposite of division is**

Students: *Multiplication*

**Correct! These are called inverse operations. These are opposite operations that undo each other. Let's write down the definition on our paper.**

*Write down the definition for inverse operations-operations that undo each other*

**So what is the opposite of adding 6?**

Students: *subtracting 6.*

**Excellent! What is the inverse operation of multiplying by 4**

Students: *dividing by 4*

**Yes, now let's look at our example of  $X - 2 = 6$ . This is an equation because it says that  $X - 2 = 6$ . An equation is a statement that says that two sides are equal. Let's write that down on our paper.**

*Write Equation- A statement that says two sides are equal*

**Great! Tell students that solving equations is all about balance so what we do to one side we must do to the other. So in our example here, what is the inverse operation of subtracting 2**

Students: Adding 2

**That's right! So when we add 2 to both sides, on the left side we get just our variable X and on the right side we get 6 + plus 2, which is 8.**

*Write down +2 on both sides and show that the answer is 8*

$$\begin{array}{r} X - 2 = 6 \\ +2 \quad +2 \\ \hline \end{array}$$

$$X = 8$$

**Now let's learn how we represent equations with drawings.**

*Draw the following equation (A) on the board:*

$$3X + 4 = 10$$

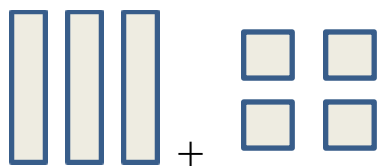
**In order to draw this equation, I need to use long rectangles for X and small squares for my numbers. So I have three X's so I'll draw three rectangles**

*Draw three rectangles like the ones below.*



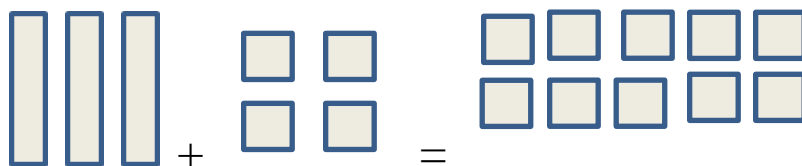
**And I'm adding four here so I'm going to draw an addition sign and four squares here**

*Draw the diagram below*



**Now to make an equation I will use the equal sign and then draw 10 squares.**

*Draw the following below*



**Now I have an equation that says  $3X + 4 = 10$ .**

*Point to each rectangle square as you say it.*

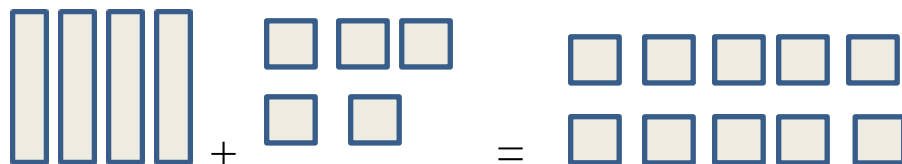
*Give the students the following two problems B and C to draw on their own (Assist students as necessary and go over the answers on the board).*

B.  $4X + 5 = 13$

C.  $5X - 10 = 15$

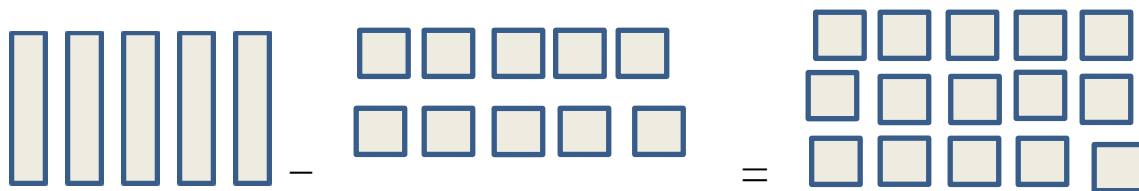
Answers:

B.





C.

**Step 2: Discuss the Strategy**

**For the next two weeks we are going to learn to solve algebra equations in a different way using some of what we've learned today. So, let's look at D together.**

*Show them the completed graphic organizer on the board (D)*

**Just like our fraction graphic organizer we have the same three sections of numerical, visual and the steps to solving**

*Point to each section as you say it.*

**Notice how we have five steps that we will use to solve an equation. First, we will draw our equations, which we have done in the visual box.**

*Point to the step and then point to the drawn equation.*

**Next, we will use inverse operations in steps two and three. First we will undo the addition or subtraction and then we will undo the multiplication or division. We will be doing this both numerically**

*Point to steps in numerical box*

**And visually**

*Point to steps in visual box*

**We will spend the next two weeks learning to make one of these. It will help you better understand equations. It will help you organize the different ways of solving equations. It will help you better remember the steps to solving equations.**

<p><b>Numerical</b></p> $3X + 2 = 11$ $\begin{array}{r} - 2 \\ \hline \end{array}$ $\frac{3X}{3} = \frac{9}{3}$ $X=3$	<p><b>Visual</b></p>	<p><b>Steps:</b></p> <ol style="list-style-type: none"> <li>1. I drew my equation.</li> <li>2. I subtracted 2 from both sides.</li> <li>3. I brought down 3X and subtracted 2 from 11 to get 9.</li> <li>4. I divided both sides by 3.</li> <li>5. 9 divided by 3 gives me three.</li> </ol>
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### Step 3: Practice Memorizing the vocabulary

**Ok, Let's review the vocabulary, what is a variable?**

Students: *A letter that stands for a number*

**Great! And what is an inverse operation**

Students: *operations that undo each other*

**Excellent! And what is an equation?**

Students: *A statement that says two sides are equal.*

**Step 4: Wrap-up:**

**Congratulate the students for doing a good job today! Tell the students today we went over some vocabulary and algebra concepts and introduced the graphic organizer. Tell them next time we will work together through the graphic organizer.**

## **Lesson Seven: Modeling each Step of the Graphic Organizer**

*(this lesson may take 1 day)*

### **Lesson Overview:**

1. The teacher will model each step to completing the graphic organizer.
2. The teacher will model self-talk and think alouds.

### **Set the Context for Learning:**

**Remind students that this week they are learning a new way to solve algebra equations. Previously, we learned and memorized vocabulary, how to draw equations, and how to do inverse operations.**

### **Step 1: Develop the Background Knowledge**

**Let's review the key vocabulary:**

**What is a variable?**

Students: *a letter that stands for a number*

**That's right a variable is a letter that stands for a number.**

**What is the inverse operation of adding?**

Students: *Subtraction*

**That's right! And what is the inverse operation of Subtraction**

Students: *Adding*

**Correct! And what's the inverse operation of multiplication**

Students: *Division*

**Excellent! And the inverse operation of division is**

Students: *Multiplication*

**Great job! And what is an equation?**

Students: *A statement that says two sides are equal*

*Ask students to draw the equation (A)  $3X + 2 = 4$ . Help students as necessary. Check student work together by asking students to share their drawings.*

## **Step 2: Discuss the Graphic Organizer**

*Show students the following completed graphic organizer (B) on the chart and remind them that they are going to learn how make one of these today by watching the teacher.*

**How many inverse operations do you notice?**

Students: *Two inverse operations*

**That's right we have subtracting two and dividing by four**

*Point to each operation as you say it*

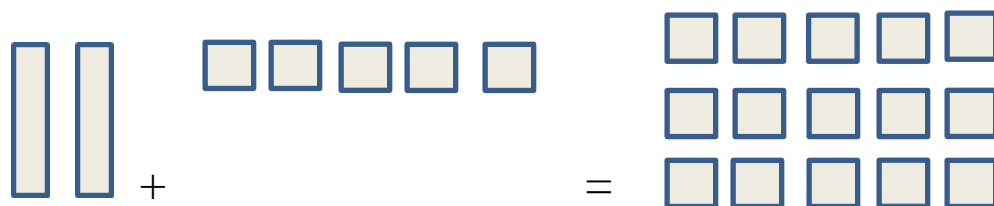
## **Step 3: Modeling the Steps to Completing the Graphic Organizer**

**Tell Students that they are now going to watch you work through an entire graphic organizer. Ask them to notice what you do for each step and the way you think about the problem. Start with the problem  $2X + 5 = 15$  in the numerical box.**

**Alright, the first thing I have to is draw my equations in the visuals box. I have to draw two rectangles to represent  $2X$  and add 5 small ones for the 5, the put 15 small ones on the side of the equal sign.**

*Match actions to words and do each step as talk through it.*

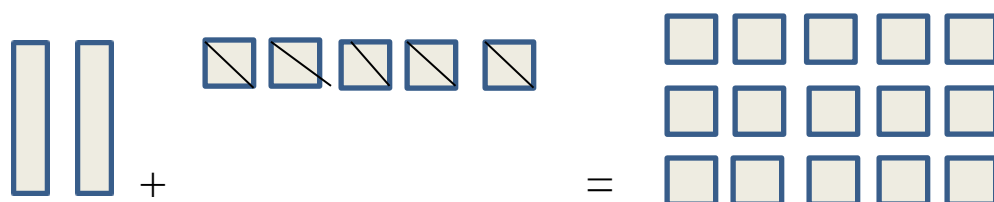
Visual square should look like this:



Now I ask myself what is the inverse operation of adding 5 and I know that the inverse of adding 5 is subtracting 5 so now I will do that to both sides in the visuals box. I will show that I am subtracting by putting slash marks on the five small squares.

*Add to the drawing as shown below.*

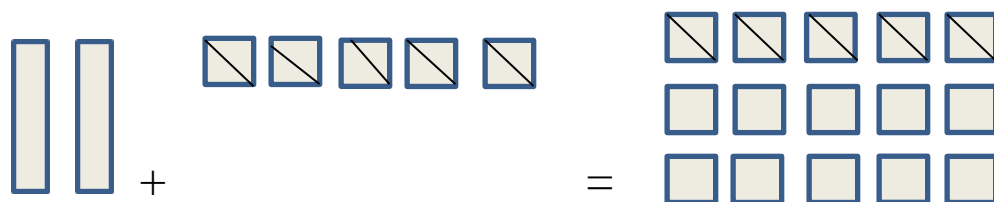
Visual square should look like this:



And what I do to one side I must do to the other so I will also subtract five squares from the 15 squares.

*Take away five squares from the other side by using slash marks*

Visual square should look like this:



So now let's show that we subtracted 5 from both sides in the numerical box as well. 5 minus 5 is zero so that leaves just 2X and 15 minus 5 is 10

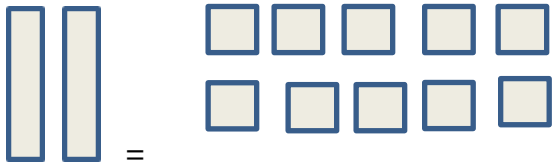
*Write the following in the numerical box.*

$$2X + 5 = 15$$

$$\begin{array}{r} -5 \\ \hline \end{array}$$

$$2X = 10$$

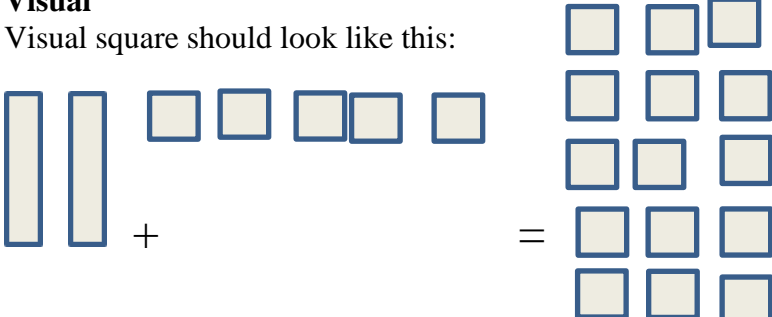
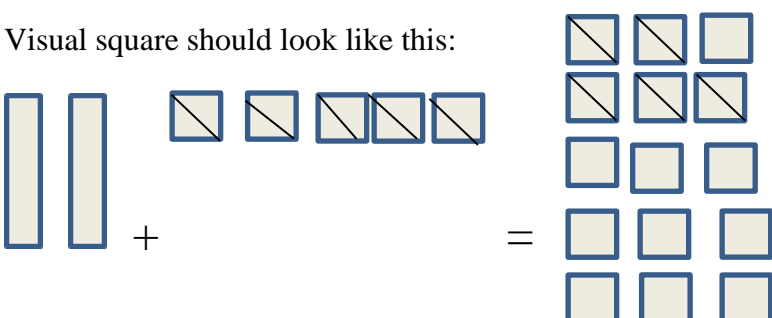
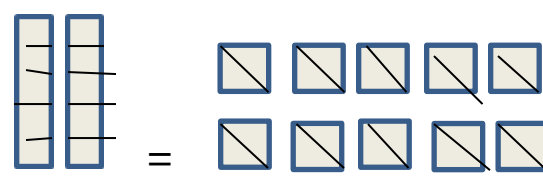
**So now that I have done the inverse operation I am left with  $2X = 10$  to solve and my drawing should match this**



**Now lets do the inverse of multiplying by 2 which in this case is dividing by 2 to both sides. So I will mark one square on each side of the equals side and see how many I have in one rectangle**

*Slash out one small square and place it in the big rectangle, then slash out another one and place it in the second one until there are none remaining*

(Graphic Organizer should look like this)

Numerical	Visual	Steps
$  \begin{array}{r}  2X + 5 = 15 \\  - \quad 5 \quad - 5 \\  \hline  2X = 10  \end{array}  $	<p>Visual square should look like this:</p> 	
	<p>Visual square should look like this:</p> 	
		

Now I need to count how many squares I have in one rectangle and in this case I have 5 so the answer is 5.

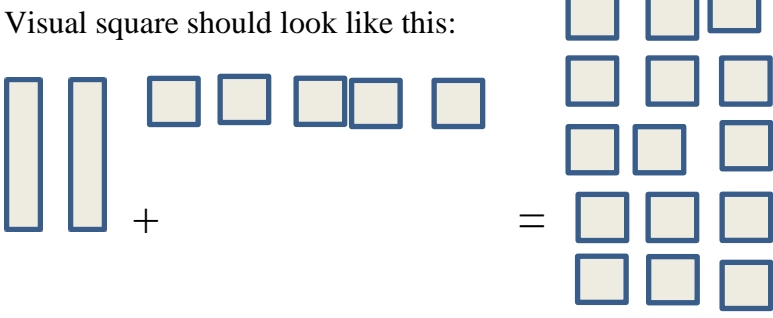
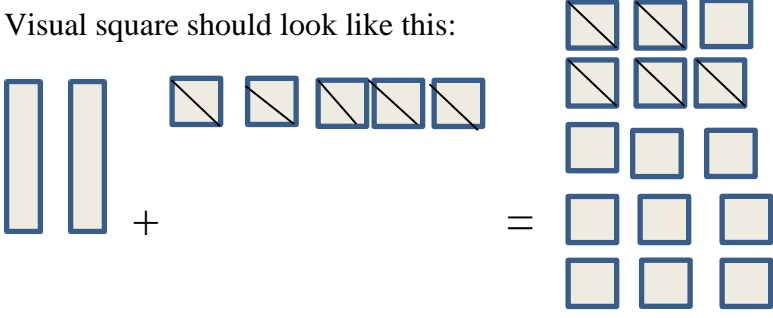
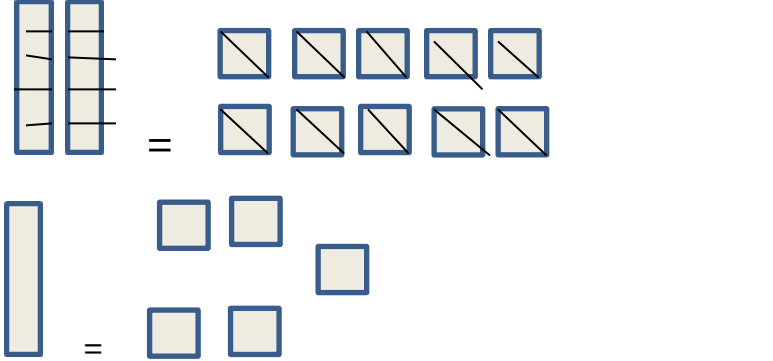
*Write down  $X=5$  in the numeric box.*

And finally I need to list my steps in the third box. So first, I drew my equations, next I subtracted 5 from both sides. Third, I brought down my 2X and subtracted 15 from 5 to get 10. Fourth, I divided both sides by 2. Fifth, I divided 10 by 2 to get 5.”

*List each step in the graphic organizer as you say it.*

Final Graphic organizer should look like this:



Numerical	Visual	Steps
$\begin{array}{r} 2X + 5 = 15 \\ - \quad 5 \quad - 5 \\ \hline 2X = 10 \end{array}$	<p>Visual square should look like this:</p>  <p>Visual square should look like this:</p>  	<ol style="list-style-type: none"> <li>1. I drew my <u>equation</u>.</li> <li>2. I <u>subtracted</u> <u>5</u> from both sides.</li> <li>3. I brought down <u>2X</u> and <u>subtracted</u> <u>5</u> from <u>15</u> to get <u>10</u>.</li> <li>4. I <u>divided</u> both sides by <u>2</u>.</li> <li>5. <u>10</u> divided by <u>2</u> gives me <u>five</u>.</li> </ol>

**Step 4: Practice memorizing the steps to adding or subtracting fractions.**

Alright, now it is your turn to practice the steps to solving equations. If I give you the problem  $3X + 2 = 11$ , can you write down the steps to solving this problem and fill in the blanks.

*Assist students as necessary to get them familiar with the steps.*

**Step 5: Wrap-up**

**Tell the students Great job today! Today you watched me work on the graphic organizer and were introduced to each step. Tell students that they practiced memorizing it today and tomorrow, you will work a problem together.**

## **Lesson Eight: Collaborative Modeling each Step of the Graphic Organizer**

*(this lesson may take 1 day)*

### **Lesson Overview:**

1. The teacher and student will work together to complete the graphic organizer.
2. The students will be practice each step to complete.

### **Set the Context for Learning:**

**Remind students that this week they are learning a new way to solve equations. Previously, we learned and memorized vocabulary, how to draw equations, and how to use inverse operations and you watched me complete the graphic organizer**

**Let's review the key vocabulary:**

**What is a variable?**

Students: *a letter that stands for a number*

**That's right a variable is a letter that stands for a number.**

**What is the inverse operation of adding?**

Students: *Subtraction*

**That's right! And what is the inverse operation of Subtraction**

Students: *Adding*

**Correct! And what's the inverse operation of multiplication**

Students: *Division*

**Excellent! And the inverse operation of division is**

Students: *Multiplication*

**Good! And what is an equation?**

Students: *A statement that says two sides are equal.*

**Step 1: Develop the Background Knowledge**

*Ask students to draw the equation (a)  $2X - 4 = 5$ . Help students as necessary. Check student work together by asking students to share their drawings.*

**Step 2: Discuss the strategy**

**What are the three steps to creating the graphic organizer.**

*Students: Numeric, Visual, and the Steps.*

**Step 3: Collaborative Modeling**

**Correct! Now let's look at this problem here.**

*Show students the following problem (B) on the board:*

$$\text{B. } 3X - 4 = 14$$

**Let's use our graphic organizer to solve this problem. First I need to write the problem down in our graphic organizer. Who can tell me which square I should put the problem in?**

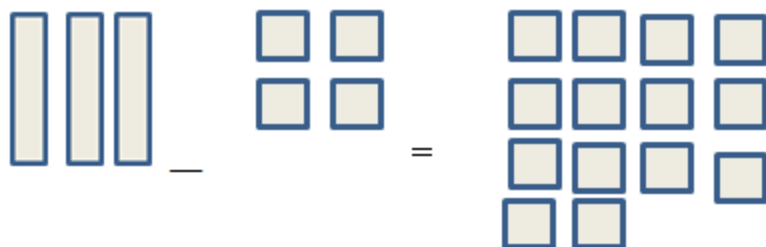
*Students: The numeric square*

**Excellent! Now who can tell me the next step to solving the problem.**

*Students: Draw the equation.*

**Right! We need to draw our equations. So I will use three long rectangles for  $3X$  and small ones for four and 14.**

*Draw the equation in the visual square*



**What is the next step to solving the problem?**

Students: *Add 4 to both sides,*

**Excellent! Yes, the inverse operation of subtracting four is adding four so I will add four squares to the right side.**

**What is minus 4 plus 4?**

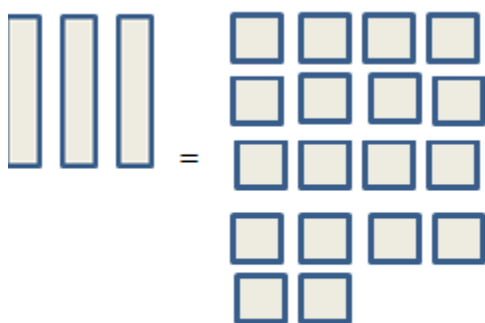
Students: *0*

**That's right! Zero so we added four to both sides which gives us what left over on the left side.**

Students: *3X or three rectangles*

**Yes, 3X is left on the left side and 18 on the right side.**

*Draw the following underneath the first equation in the visuals box*



Excellent, Now let's show that in our numerical square as well.

*Draw the following in the numerical square*

$$\begin{array}{r}
 3X - 4 = 14 \\
 \downarrow \quad \begin{array}{r} + 4 \\ \hline \end{array} \quad \begin{array}{r} + 4 \\ \hline \end{array} \\
 0 \qquad \qquad 18 \\
 3X = 18
 \end{array}$$

**What do I need to do next?**

Students: *divide by 3 on both sides.*

*Perform the division in the visuals square,  
then the numeric square and ask students to help you*

**Now let's write that numerically:**

*In the numerical box write down*

$$\begin{array}{r}
 \underline{3X} = \underline{18} \\
 3 \quad 3 \\
 X = 6
 \end{array}$$

Final answer should be  $X = 6$ .

**So, for our first step we drew our ?**

Students: *Equations*

**Then we**

Students: *added 4 to both sides*

**Next we brought down the**

Students: 3X

**And**

Students: *added 4 to 14 to get 18*

**Fourth, we**

Students: *divided*

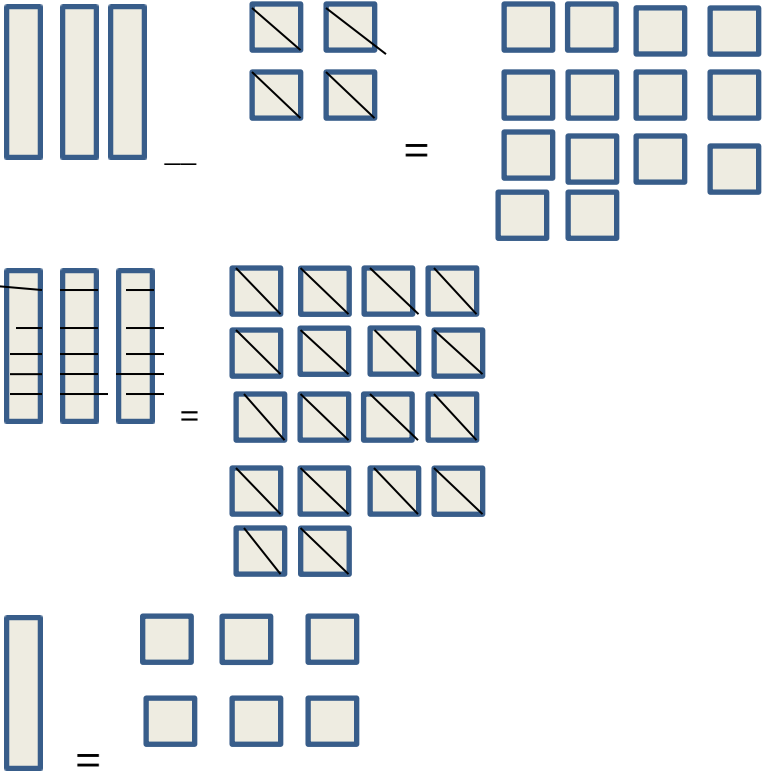
**Both sides by**

Students: 3

**18 divided by 3 gives us**

Students: 6

*Write them down in the Steps square as they say each one.* (Complete graphic organizer is shown below)

Numerical	Visual	Steps
$3X - 4 = 14$ $\underline{\quad + 4 \quad} \quad \underline{\quad + 4 \quad}$ $\frac{3X}{3} = \frac{18}{3}$ $X = 6$		<ol style="list-style-type: none"> <li>1. I drew my equation.</li> <li>2. I added 4 to both sides.</li> <li>3. I brought down 3X and added 4 to 14 to get 18.</li> <li>4. I divided both sides by 3.</li> <li>5. 18 divided by 3 gives me six.</li> </ol>

#### Step 4: Memorize the strategy

*Take down the graphic organizer so students have no hints and ask them to write them the steps to solving the following equation (B)  $2X - 4 = 12$*

#### Step 5: Wrap-up

**Tell students great job today! Today we worked together at completing a graphic organizer and tomorrow students will work independently at completing the graphic organizer.**



## **Lesson Nine: Independent Practice for completing the Graphic Organizer or Guide Practice**

*(this lesson may take 1 day)*

### **Lesson Overview:**

6. The students will complete the graphic organizer independently.

### **Set the Context for Learning:**

**Remind students that this week they are learning a new way to solve equations. Previously, we worked together to complete the graphic organizer. Today, we will split into groups with some of you working independently to complete one and others working in pairs with the teacher.**

### **Step 1: Develop the Background Knowledge**

**What are the three steps to creating the graphic organizer.**

Students: *Numeric, Visual, and the Steps.*

### **Step 2: Discuss the strategy**

**Write the following problem on the board  $3X - 4 = 16$ . What steps would you take to solve this problem?**

Students: *Draw our equations. Add four to both sides. Divide by three.*

### **Step 3: Guided Support or Independent Practice**

Depending on the memorization quizzes the previous day. Divide the students into two groups. Those who remembered all the steps to solving the problem will work independently on completing a graphic organizer and those who did not will work in pairs and/or with the teacher to practice some more problems.

Group one is given the following problem:  $4X - 5 = 19 = ?$

Group 2 works in pairs and with guided support to solve the following problem:  $6X - 4 = 8$   
(See answer sheet for complete graphic organizers filled out)

### **Step 4: Memorize the steps:**

Clear away all notes and ask students to write down the steps to solve the equation (B)  $5X + 5 = 15$ .

**Step 5: Wrap- up:**

**Tell students good job today! Today we filled out graphic organizers independently and tomorrow we will practice solving equations without graphic organizers.**

## Lesson Ten: Solving Equations Practice

*(this lesson may take 1 day)*

### Lesson Overview:

7. The students will independently solve three equations.

### Set the Context for Learning:

**Remind students that this week they are learning a new way to solve equations. They have been practicing completing graphic organizers and today they will just solve equations without the graphic organizer.**

### Step 1: Develop the Background Knowledge

**What are the three steps to creating the graphic organizer.**

Students: *Numeric, Visual, and the Steps.*

### Step 2: Discuss the strategy

**Write the following problem on the board  $2X + 4 = 12$ . What steps would you take to solve this problem?**

Students: *Draw our equations. Subtract four from both sides. Divide by two.*

### Step 3: Independent Practice

Have students complete the following three equations on the student sheets (Walk around and monitor their progress). They can solve with numbers or drawings at this point.

A.  $2X + 4 = 10$

B.  $3X - 4 = 17$

C.  $5X + 3 = 23$

### Step 4: Wrap-Up

**Tell students Good job today! Today, we worked independently to solve the equations. Tomorrow they will take the fraction and algebra posttests**

**Appendix E: Sample Fraction Probe****Fraction Quiz 1**

Name: \_\_\_\_\_

Date: \_\_\_\_\_

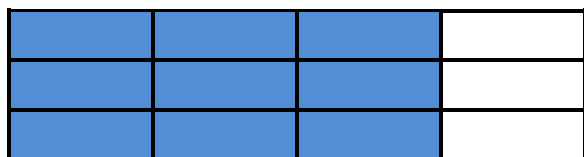
**Part One: Answer the following questions**

1. Name this fraction.



Answer: \_\_\_\_\_

2. Are these two fractions equivalent? Circle Yes or No



Justify your reasoning by drawing each fraction:

3. Draw the fraction  $\frac{2}{5}$ .

4. Are the fractions  $\frac{1}{4}$  and  $\frac{2}{8}$  equivalent? Circle Yes or No

(Justify your reasoning)

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**Part Two: Use fraction computation to solve each problem.**

5.  $\frac{1}{2} + \frac{1}{3} =$

Answer: \_\_\_\_\_

6.  $\frac{2}{4} + \frac{1}{5} =$

Answer: \_\_\_\_\_

7.  $\frac{3}{4} - \frac{1}{2} =$

Answer: \_\_\_\_\_

8.  $\frac{3}{5} - \frac{1}{2} =$

Answer: \_\_\_\_\_





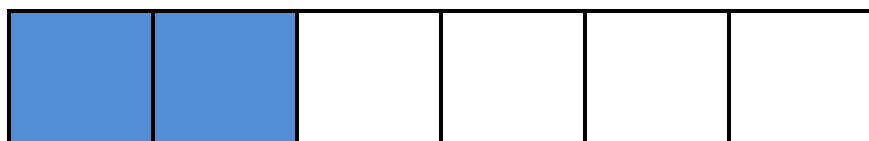
**Fraction Quiz 2**

Name: \_\_\_\_\_

Date: \_\_\_\_\_

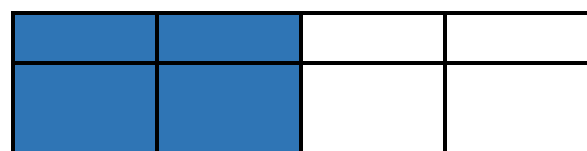
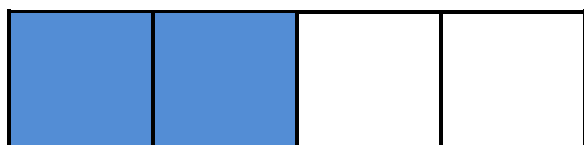
**Part One: Answer the following questions**

1. Name this fraction.



Answer: \_\_\_\_\_

2. Are these two fractions equivalent? Circle Yes or No.



Justify your reasoning:

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:

3. Draw the fraction  $\frac{4}{5}$ .

4. Are the fractions  $\frac{1}{5}$  and  $\frac{2}{10}$  equivalent? Circle Yes or No.

(Justify your reasoning by drawing each fraction)

**Part Two: Use fraction computation to solve each problem.**

5.  $\frac{1}{4} + \frac{1}{3} =$

Answer: \_\_\_\_\_

$$6. \frac{2}{4} + \frac{4}{5} =$$

Answer: \_\_\_\_\_

$$7. \frac{3}{4} - \frac{2}{5} =$$

Answer: \_\_\_\_\_

8.  $\frac{2}{3} - \frac{2}{5} =$

Answer: \_\_\_\_\_

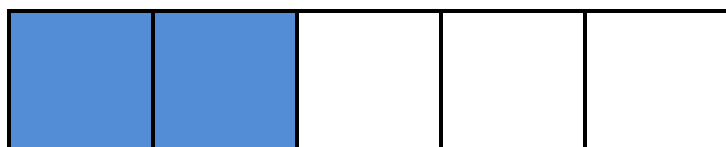
**Fraction Quiz 3**

Name: \_\_\_\_\_

Date: \_\_\_\_\_

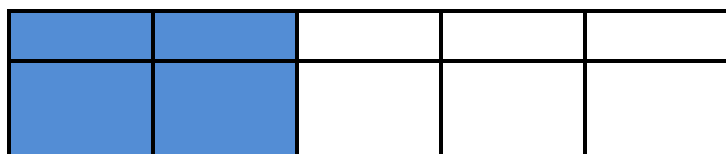
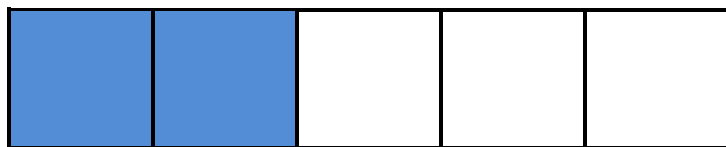
**Part One: Answer the following questions**

1. Name this fraction.



Answer: \_\_\_\_\_

2. Are these two fractions equivalent? Circle Yes or No.



Justify your reasoning:

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3. Draw the fraction  $\frac{3}{8}$ .

4. Are the fractions  $\frac{1}{2}$  and  $\frac{6}{8}$  equivalent? Circle Yes or No.

(Justify your reasoning by drawing each fraction)

**Part Two: Use fraction computation to solve each problem.**

5.  $\frac{1}{2} + \frac{1}{5} =$

Answer: \_\_\_\_\_

6.  $\frac{1}{5} + \frac{1}{3} =$

Answer: \_\_\_\_\_

7.  $\frac{1}{2} - \frac{1}{4} =$

Answer: \_\_\_\_\_

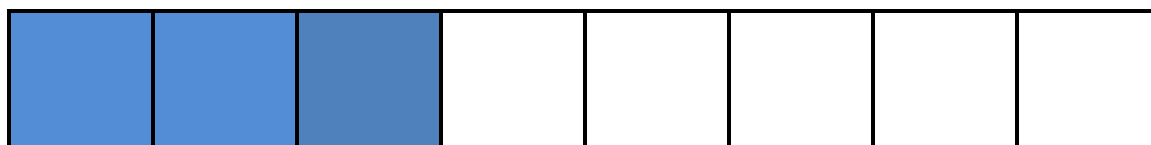
8.  $\frac{1}{2} - \frac{2}{5} =$

Answer: \_\_\_\_\_



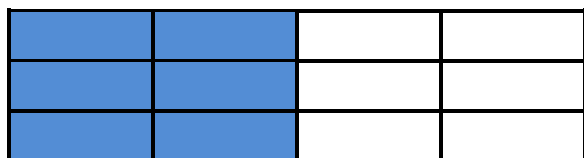
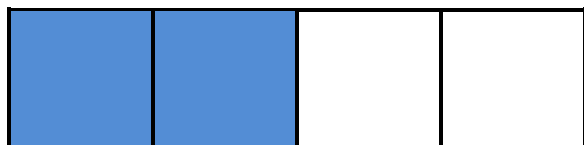
**Fraction Quiz 4****Name:** \_\_\_\_\_**Date:** \_\_\_\_\_**Part One: Answer the following questions**

1. Name this fraction.



Answer: \_\_\_\_\_

2. Are these two fractions equivalent? Circle Yes or No.



Justify your reasoning:

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3. Draw the fraction  $\frac{1}{6}$ .

4. Are the fractions  $\frac{1}{3}$  and  $\frac{3}{6}$  equivalent? Circle Yes or No.

(Justify your reasoning by drawing each fraction)

**Part Two: Use fraction computation to solve each problem.**

5.  $\frac{1}{4} + \frac{2}{3} =$

Answer: \_\_\_\_\_

6.  $\frac{1}{2} + \frac{3}{5} =$

Answer: \_\_\_\_\_

7.  $\frac{3}{4} - \frac{1}{5} =$

Answer: \_\_\_\_\_

8.  $\frac{3}{4} - \frac{1}{3} =$

Answer: \_\_\_\_\_

**Appendix F: Sample Algebra Probe****Algebra Quiz 1****Name:** \_\_\_\_\_**Date:** \_\_\_\_\_**Part One: Answer the following questions**

1. Draw the equation  $2X + 3 = 5$

2. Draw the equation  $4X - 5 = 10$

3.  $2X + 4 = 20$ , what would be the first step to solving this equation?

4.  $2X + 4 = 20$ , What would be the second step to solving this equation?

**Part Two: Solve each Equation**

5.  $2x + 5 = 15$

Answer: \_\_\_\_\_

6.  $3x + 3 = 21$

Answer: \_\_\_\_\_

7.  $3x - 1 = 14$

Answer:

---

8.  $6x - 4 = 8$

Answer:\_\_\_\_\_

## Algebra Quiz 2

**Name:**\_\_\_\_\_

**Date:**\_\_\_\_\_

**Part One: Answer the following questions**

1. Draw the equation  $3X + 4 = 10$
2. Draw the equation  $2X - 6 = 8$
3.  $3X + 6 = 18$ , what would be the first step to solving this equation?



4.  $3X + 6 = 18$ , What would be the second step to solving this equation?

**Part Two: Solve each Equation**

5.  $5x + 1 = 6$

Answer: \_\_\_\_\_

6.  $7x + 2 = 23$

Answer: \_\_\_\_\_

7.  $4x - 1 = 15$

Answer:

---

8.  $4x - 2 = 18$

Answer:\_\_\_\_\_

## Algebra Quiz 3

**Name:** \_\_\_\_\_

Date: \_\_\_\_\_

**Part One: Answer the following questions**

1. Draw the equation  $4X + 5 = 9$
2. Draw the equation  $3X - 8 = 10$
3.  $4X + 2 = 18$ , what would be the first step to solving this equation?

4.  $4X + 2 = 18$ , What would be the second step to solving this equation?

**Part Two: Solve each Equation**

5.  $2x + 6 = 14$

Answer: \_\_\_\_\_

6.  $2x + 9 = 27$

Answer: \_\_\_\_\_

7.  $3x - 4 = 11$

Answer:

---

8.  $6x - 4 = 14$

Answer: \_\_\_\_\_

## Algebra Quiz 4

**Name:** \_\_\_\_\_

Date: \_\_\_\_\_

**Part One: Answer the following questions**

1. Draw the equation  $3X + 4 = 8$
2. Draw the equation  $2X - 5 = 12$
3.  $5X + 15 = 20$ , what would be the first step to solving this equation?

4.  $5X + 15 = 20$ , What would be the second step to solving this equation?

**Part Two: Solve each Equation**

5.  $3x + 7 = 16$

Answer: \_\_\_\_\_

6.  $4x + 5 = 25$

Answer: \_\_\_\_\_

7.  $2x - 4 = 16$

Answer:

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8.  $4x - 6 = 10$

Answer: \_\_\_\_\_



9.  $8 = 2X + 4$

Answer:

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## Appendix G: Fidelity Adherence Form

Lesson: \_\_\_\_\_

Date: \_\_\_\_\_

Total Minutes of Instruction: \_\_\_\_\_

Teacher: \_\_\_\_\_

**Please check if each planned step was completed and place an X if a step was skipped/omitted. Write NA if you did not get to a step for unforeseen circumstances and will finish it during the next class period (e.g., class ended early, major behavior incident).**

**1. Develop the background knowledge:** Minutes \_\_\_\_\_

- a. Reviewed the Key vocabulary \_\_\_\_\_
- b. Reviewed the steps to the graphic organizer \_\_\_\_\_
- c. Reviewed how to draw fractions \_\_\_\_\_

**2. Discussed the strategy:** Minutes \_\_\_\_\_

- a. Discussed the graphic organizer with class \_\_\_\_\_
- b. Discussed benefits to using it \_\_\_\_\_
- c. Emphasized that there were different denominators \_\_\_\_\_
- d. Discussed all three sections of the graphic organizer \_\_\_\_\_

**3. Modeled the strategy:** Minutes \_\_\_\_\_

- a. Modeled how to draw each fraction or equation \_\_\_\_\_
- b. Modeled how to figure out how many horizontal lines to draw of to use inverse operations \_\_\_\_\_
- c. Modeled how to figure out equivalent fractions \_\_\_\_\_

or use the second inverse operation \_\_\_\_\_

d. Modeled how to add or subtract to find the answer  
or the final answer to the equations \_\_\_\_\_

e. Modeled how to write down each step for solving  
the problem \_\_\_\_\_

f. Mentioned how the steps spell FOPS \_\_\_\_\_

4. **Memorize it:** Minutes \_\_\_\_\_

a. Allowed students to practice memorizing either the  
Vocabulary or steps to solve it \_\_\_\_\_

b. Had students independently write down steps \_\_\_\_\_

5. **Collaborative practice:** Minutes \_\_\_\_\_

a. Guided students to draw each fraction or equation \_\_\_\_\_

b. Guided students to figure out how many horizontal lines to  
draw of to use inverse operations \_\_\_\_\_

c. Guided students to figure out equivalent fractions  
or use the second inverse operation \_\_\_\_\_

c. Guided students to add or subtract to find the answer  
or the final answer to the equations \_\_\_\_\_

d. Guided students to write down each step for solving  
the problem \_\_\_\_\_

6. **Independent practice or Supported practice:** Minutes \_\_\_\_\_

a. Gave students a similar problem to work or  
Worked in small groups with students \_\_\_\_\_

**7. Wrap- Up:**

Minutes \_\_\_\_\_

- a. Reminded students of today's lesson \_\_\_\_\_
- b. Congratulated students on a good job \_\_\_\_\_
- c. Stated the objective for the next lesson \_\_\_\_\_

## Appendix H: Quality of Instruction Fidelity Sheet

### Quality of Instruction

Category	Definition	Rating
Rate the effectiveness of the teacher in using the math vocabulary from the graphic organizer instruction.	1= Teacher rarely uses the math vocabulary terms. 2= Teacher sometimes uses the math vocabulary terms. 3= Teacher often uses the math vocabulary terms.	
Rate the effectiveness of the teacher in performing the math correctly.	1= Teacher makes multiple math errors without correcting. 2= Teacher makes one or two math errors without correcting. 3= Teacher makes no math errors or makes errors but self-corrects.	
Rate the effectiveness of the teacher in calling on a variety students.	1= Teacher calls on the same student to answer questions when teaching the graphic organizer. 2= Teacher calls on the same two students when teaching the graphic organizer. 3= Teacher calls on more than two students when teaching the graphic organizer.	

## Appendix I: TARF Teacher Questionnaire

## TARF-R Teacher FORM

### Graphic Organizer

**Treatment Acceptability Rating Form – Revised (TARF-R: Reimers & Wacker, 1988)**  
**Modified for the Using a Using a Graphic Organizer to Increase Math Performance for**  
**Students**  
**Teacher Form**

**Student:** \_\_\_\_\_ **Date:** \_\_\_\_\_

**Directions: Please complete the items listed below as they pertain ONLY to the graphic organizer for each student. These items should be completed by placing a check mark on the line under the question that best indicates how you feel about the use of this math strategy.**

1. How clear is your understanding of the graphic organizer instruction procedures?

Not at all clear Neutral Very clear

2. How acceptable do you find graphic organizer instruction to be for you and your students?

Not at all \_\_\_\_\_ Neutral \_\_\_\_\_ Very

3. How willing are you to use the graphic organizer in the future?

Not at all \_\_\_\_\_ Neutral \_\_\_\_\_ Very

4. Given your student math problems, how reasonable do you find the graphic organizer strategy?

\_\_\_\_\_

Not at all

\_\_\_\_\_

Neutral

\_\_\_\_\_

Very

5. To what extent do you think there might be disadvantages in using the graphic organizer?

\_\_\_\_\_

Not at all

\_\_\_\_\_

Neutral

\_\_\_\_\_

Many are  
likely

6. How likely is graphic organizer instruction to make permanent improvements in your students' academic performance?

\_\_\_\_\_

Unlikely

\_\_\_\_\_

Neutral

\_\_\_\_\_

Very likely

7. How much time will be needed each day for you to carry out graphic organizer instruction?

\_\_\_\_\_

Little time  
needed

\_\_\_\_\_

Neutral

\_\_\_\_\_

Much time  
will be needed

8. How confident are you that graphic organizer strategies were effective?

\_\_\_\_\_

Not at all

\_\_\_\_\_

Neutral

\_\_\_\_\_

Very  
confident

9. Compared to other students who struggle with math, how serious are your students' problems in your classroom?

\_\_\_\_\_

Not at all  
Serious

\_\_\_\_\_

Neutral

\_\_\_\_\_

Very serious

10. How disruptive will it be to your classroom (in general) to use graphic organizer instruction.?

\_\_\_\_\_  
 Not at all                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Neutral                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Very disruptive

11. How effective is graphic organizer instruction for you?

\_\_\_\_\_  
 Not at all                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Neutral                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Very effective

12. How affordable is graphic organizer instruction for your classroom?

\_\_\_\_\_  
 Not at all                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Neutral                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Very affordable

13. How much do you like the procedures in the graphic organizer?

\_\_\_\_\_  
 Not like                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_  
 them at all                      \_\_\_\_\_  
 Neutral                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Like them  
 very much

14. How willing will are you to work with other teachers on graphic organizer instruction?

\_\_\_\_\_  
 Not at all                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Neutral                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Very willing

15. To what extent are undesirable side-effects likely to result from graphic organizer instruction?

\_\_\_\_\_  
 Not likely                      \_\_\_\_\_                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Neutral                      \_\_\_\_\_                      \_\_\_\_\_  
 \_\_\_\_\_  
 Many side-effects are likely

16. How much discomfort is the student likely to experience during graphic organizer instruction?



\_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_  
 No discomfort      Neutral      Very much  
 at all      discomfort

17. How severe are your students' math difficulties in your classroom?

\_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_  
 Not at all      Neutral      Very severe  
 severe

18. How well would graphic organizer fit into your classroom curriculum?

\_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_  
 Not at all      Neutral      Very well  
 Well

19. How willing would you be to change your classroom routine to use graphic organizer instructional strategies?

\_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_  
 Not at all      Neutral      Very willing  
 Willing

20. How well does the graphic organizer instruction fit within you school curriculum?

\_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_      \_\_\_\_\_  
 Not at all      Neutral      Very well

## Appendix J: TARF Student Questionnaire

### TARF-R Student FORM Graphic Organizer

#### **Treatment Acceptability Rating Form – Revised (TARF-R: Reimers & Wacker, 1988) Modified for the Using a Using a Graphic Organizer to Increase Math Performance for Students Teacher Form**

**Student:** \_\_\_\_\_ **Date:** \_\_\_\_\_

**Directions: Please complete the items listed below. Put a check on the line that best shows how you feel.**

1. How clear is your understanding of the study?

_____	_____	_____	_____	_____	_____	_____
Not at all clear			Neutral			Very clear

2. How acceptable do you find graphic organizer to be for you?

_____	_____	_____	_____	_____	_____	_____
Not at all			Neutral			Very helpful

3. How helpful did you find the graphic organizer to your wanting to join in class?

_____	_____	_____	_____	_____	_____	_____
Not at all			Neutral			Very

4. How willing are you to use the graphic organizer in the future?

_____	_____	_____	_____	_____	_____	_____
Not at all			Neutral			Very

5. How reasonable do you find the graphic organizer strategy?

Not at all Neutral Very

6. Do you think there might be problems in using the graphic organizer?

Not at all    Neutral    Many are likely

7. How likely is graphic organizer instruction to make long term improvements in your math?

Unlikely Neutral Very likely

8. How confident are you that graphic organizer strategies helped you?

Not at all				Neutral				Very confident
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9. How much do you like the steps in the graphic organizer?

Not like them at all      Neutral      Like them very much

10. How much do you think other students liked using the graphic organizer?

Not at all                  a little                  Like it alot

## **Appendix K: Student Interview Questions about the Graphic Organizer**

### **Student Interview Questions:**

1. What did you like about using the graphic organizer?
2. Tell me which square was your favorite and why.
3. Which square do you think helped you the most and Why?
4. Do think there is a difference between fractions and algebra?
5. Why do you think that?
6. Do you think you need to use all three squares and why?
7. How did you feel when we were about to start a math lesson?

## **Appendix L: Teacher Interview Questions about the Graphic Organizer**

### **1. Teacher Interview Questions**

1. What did you like about teaching the graphic organizer?
2. Which square you feel benefited the students the most and why?
3. Which square did you find the easiest and hardest to teach and why?
4. Did you find one graphic organizer easier to use than another? Why?
5. Do you think you need to teach all three squares and why?
6. Is there a difference between teaching fractions and algebra to you? To your students?
7. Why do you think that?
8. What did you like about scripted lessons?
9. What did you not like about scripted lessons?
10. What improvements would you make to the instruction?
11. What improvements would you make to the lesson plans?